**WEST-EAST HIGH SPEED FLOW FIELD CONFERENCE**
**19-22, November 2007**
**Moscow, Russia**

# A FORMATION OF TURBULENCE SPECTRUM IN FREE SHEAR FLOW

**Oleg M. Belotserkovskii* Valery M. Chechetkin** and Alexei M. Oparin***

** Institute for Computer Aided Design*
*Russian Academy of Sciences*
*Vtoraya Brestskaya st. 19/18, 123056 Moscow, Russia*
*Emails: o.bel@icad.org.ru, a.oparin@icad.org.ru*
*** Keldysh Institute of Applied Mathematics*
*Russian Academy of Sciences*
*Miusskaya sq. 4, 125047 Moscow, Russia*
*Email: chech@gin.keldysh.ru*

**Key words:** free shear flow, instability, turbulence, numerical modeling, accretion disk.

**Abstract.** We propose new physical model of the development of turbulence in free shear flows which contains main idea about leading role of large scale structures. Our theory is based on the results of numerical simulations of turbulent flow development. In these calculations the main idea of the proposed theory of turbulence in free shear flow is stated as follow: the onset of turbulence begins with the formation of large vortices. Very important moment is the appearance of the secondary instability. This instability leads to the creation of turbulence cascade. In our calculation we obtain the spectrum which is different from Kolmogorov's universal spectrum. The spectrum is sensitive to the problem dimensionality. We show our results of the evolution of large-scale turbulence in accretion disks for thin accretion disk. A possible mechanism explaining the transfer of angular momentum by large vortices that form in the disk without any noticeable heating of the matter is suggested.

## 1. INTRODUCTION

The turbulence is mystery of XX century. Comprehensive turbulence theory has not been framed until present time.

In 1985, O.M. Belotserkovskii suggested that spectrum of fully developed free turbulence has been fell to independent pieces. Then the turbulence could be simulated without using any subgrid-scale model and adjusting any semi-empirical constants [1]. The review of works on this approach can be found in [2,3], where it was applied to free turbulent flows behind moving bodies (including both near- and far-wake flow structures), oceanic flows, Taylor-Couette flow, evolution of turbulent mixing zones, and other important problems concerning the onset of turbulence. The approach reflects the multidimensional and unsteady nature of the flows in question and takes into account phenomena related to compressibility, as well as effects due to viscosity (dominated by molecular mechanism). In those studies, it was also shown that large-scale vortices play a dominant role in turbulent flow structure and small-scale eddies followed from large structure.

The basic ideas of direct numerical simulation of turbulence rely on the following two hypotheses supported by experimental evidence:

(1) large-scale coherent vortices and small-scale stochastic turbulence statistically independent at high Reynolds numbers;

(2) molecular viscosity (more generally, the mechanism of energy dissipation) plays a minor role in the analysis of large-scale vortex dynamics.

Large vortices carry the greater part of the energy of turbulent motion and determine the flow structure. But kinetic energy of large vortices does not dissipate into heat. Dissipation of kinetic energy of turbulence is connected with small-scale turbulence. The dynamics of large vortices does not reflect the structure of random fluctuations, being governed by the Navier-Stokes equations in which the inertial terms dominate over the viscous terms. Accordingly, the structure of a vortex develops as a result of combined action of pressure gradients and transient forces arising from velocity fields. Therefore, both formation of large-scale vortices and ensuing flow structure must be described by the Euler equations.

Very important aspect for turbulence is a spectrum problem. This problem is connected with physical processes, which lead to creation of various scales of turbulence. In beginning we investigate the development of turbulence for 2D shear flow. For such flow the appearance of small-scale part of spectrum is connected with the interaction between large vortices with main flow and/or boundaries. For 3D shear flow the turbulence spectrum is defined by secondary instability. Secondary instability is connected with surface instability of large-scale vortices.

The role of large-scale turbulence is very important in modern astrophysics. In our paper we shall discuss transport of angular moment for accretion disk.

In this paper, we consider the problem of formation and evolution of large-scale turbulence flow from initially small disturbances. The problem is of significant interest as regards various disk flows under astrophysical conditions [8-11]. Formation of a large-scale turbulence makes it possible for angular momentum to be transferred by large turbulent structures that form in shear flow in accretion disk. Transfer of angular momentum by large turbulent vortices does not result in any noticeable heating of the matter. A process like this provides the required velocity of accretion accompanied by a comparatively low temperature of the accretion disk. Thus, a new mechanism is suggested for transfer of angular momentum in accretion disks, which yields accretion characterized by smaller local heat emission.

## 2. 2D MODELING OF FREE SHEAR FLOWS

To examine physical scenarios of the onset of turbulence, we performed an extensive series of numerical simulations of free shear flows of an ideal compressible gas.

For simulations we used two-dimensional Euler gas dynamics equations in Cartesian coordinates:

$$\frac{\partial \rho}{\partial t} + \nabla_i \left( \rho u_i \right) = 0,$$

$$\frac{\partial \left( \rho u_i \right)}{\partial t} + \nabla_j \left( \rho u_i u_j \right) = -\frac{\partial P}{\partial x_i} + \rho g_i, \; i = 1, 2,$$

$$\frac{\partial(\rho E)}{\partial t} + \nabla_j \left( (\rho E + P) u_j \right) = \rho g_i u_i,$$

$$\frac{\partial(\rho C)}{\partial t} + \nabla_i (\rho C u_i) = 0.$$

The ideal gas law is used in the following form:

$$P = (\gamma - 1)\rho\varepsilon .$$

Here $t$ is the time, $\rho$ is the gas density, $P$ is the pressure, $\varepsilon$ is the specific internal energy, $E = \varepsilon + u_i u_i$ is the full specific energy, $\gamma$ is the ratio of specific heats, $u_i$ is the component of the gas velocity, $g_i$ is the component of the gravitational acceleration, $C$ is the virtual concentration for visualization.

We used monotonic dissipative stable finite-difference schemes with positive operators [1-3].

We analyzed the evolution of a shear layer with a uniform velocity gradient. Figure 1 demonstrates that large vortices of diameter comparable to the shear-layer thickness develop first. The vortex motion in a finite volume is generated by a pressure gradient. The shear layer breaks up into large vortices, and smaller eddies develop in their wakes. At the final instant of the simulation, the flow consists of a single vortex occupying the computational domain. This effect is explained by the attraction of vortices with similar vorticity signs due to the Zhoukovskij force (Figure 2). The computation was performed with free-flow conditions set on the upper and lower boundaries combined with periodic conditions at the upstream and downstream boundaries.

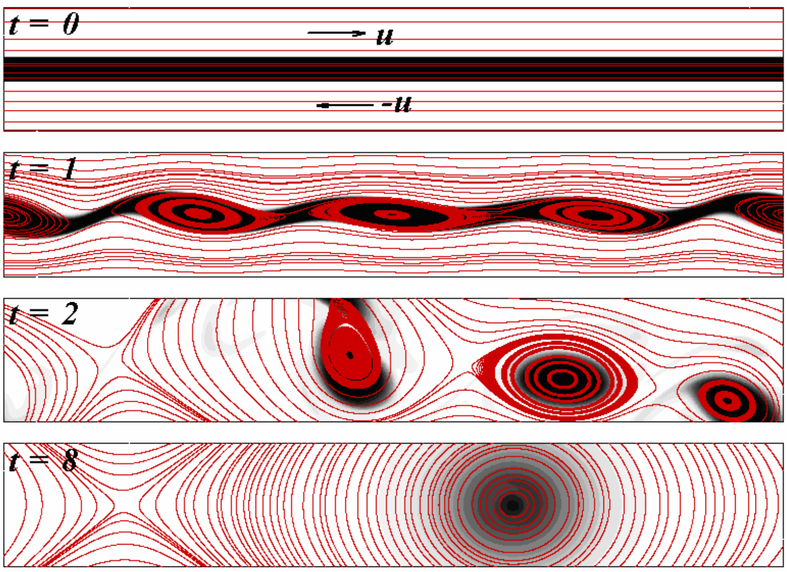


Figure 1. Development of large-scale vortices in a free turbulent shear layer. Streamlines are shown at instants separated by equal time intervals, including the starting moment. The grayscale value represents the concentration of particles initially localized in the shear layer.

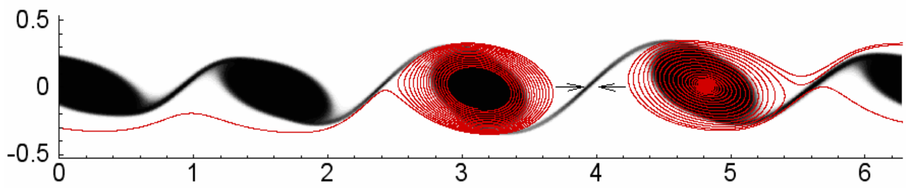


Figure 2. Attraction of vortices with similar vorticity signs.

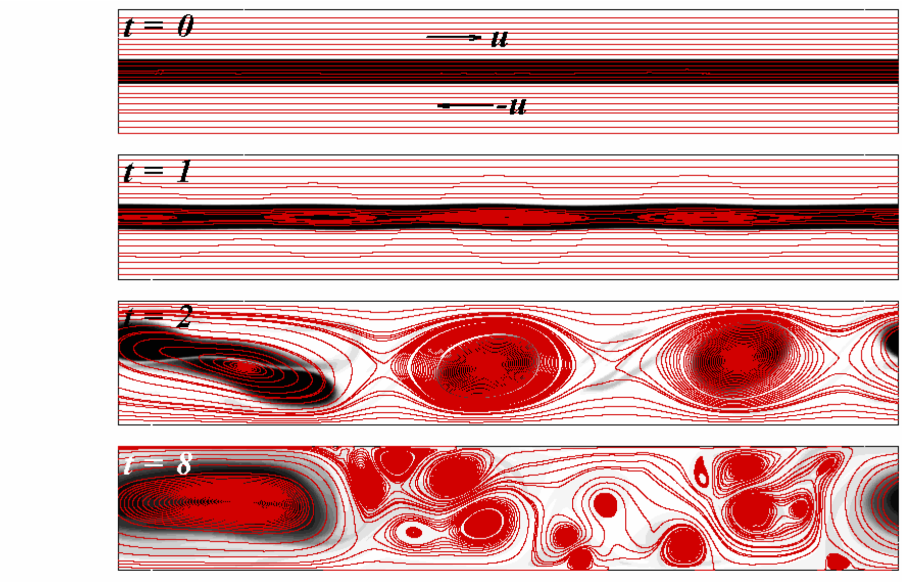


Figure 3. Flow analogous to that shown in Figure 1, but confined between walls.

Figure 3 shows the results obtained by computing the evolution of turbulence in a similar shear layer, but with impermeability conditions set on the upper and lower boundaries. Here, the onset of turbulence follows the scenario observed in the preceding simulation. However, the final turbulent flow has a complicated pattern involving both large-scale vortices and smaller structural elements. The turbulence spectrum is due to interactions of the background flow with walls and large-scale vortices.

We have studied Taylor-Couette flow in two-dimensional case. Inviscid Couette flow is taken as an initial flow. For investigation of stability of such flow we used the procedure of introducing of perturbation of radial velocity of small amplitude and definite frequency. Results of computations are given on figure 4. Birth of large vortices characterizes the initial stage of instability. Then, due to large vortices interaction several vortices remain in the end of simulation. We check the dependence of our results from size of grid. The results for three different grids (sizes are marked in the center) are presented in Figure 4. Vorticity fields are shown for final time moment. The sizes of grid are pointed. If the grid becomes more detailed then the numerical viscosity decreased and, correspondingly, numerical Reynolds number increased. The measured in numerical experiments values are $\mathrm{Re}_{num} = 1.2\cdot 10^5, 1.5\cdot 10^5, 1.8\cdot 10^5$. The energy of developed turbulent motion depends slightly from size of grid. The appropriate time dependences are given in lower right part of Figure 4.

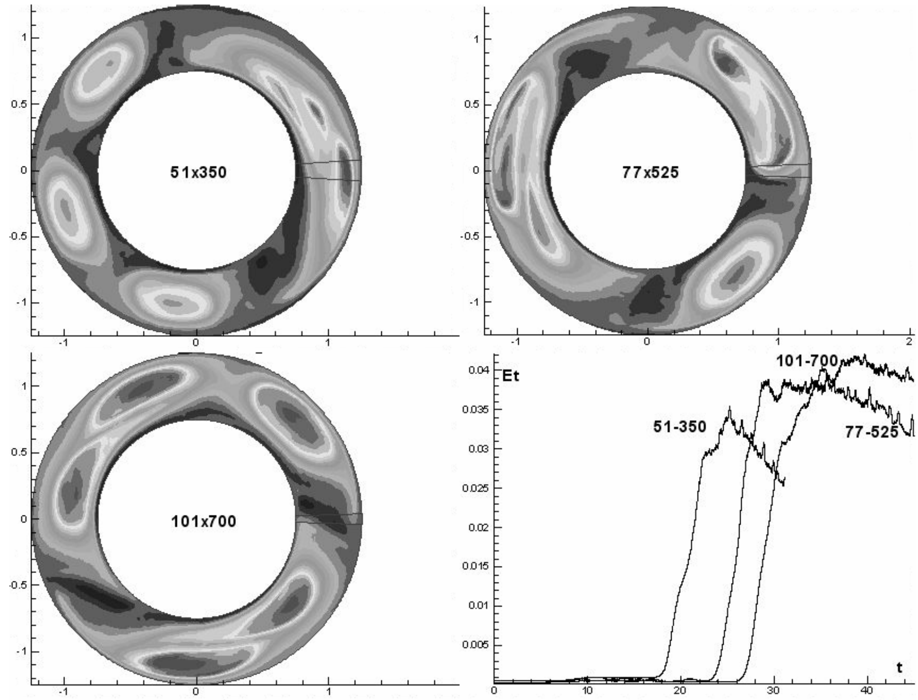


Figure 4. Vorticity fields are give for three different grids. In lower right part the appropriate time dependences of developed turbulent energy are presented.

## 3. 3D MODELING OF FREE SHEAR FLOWS

To investigate physical scenario of the onset of turbulence, we performed numerical simulations of free shear flows of an ideal compressible gas.

For simulations we used three-dimensional Euler gas dynamics equations in Cartesian coordinates with ideal gas law.

Let is consider the flow of matter in the integration domain of ($0 \le x \le L_x$; $0 \le y \le L_y$; $-L_z/2 \le z \le L_z/2$). Initial velocity along *x*-direction ($u_1$) is used in the following form:

$$u_1 = u_0,\ H/2 \le z \le L_z/2,$$

$$u_1 = -u_0,\ -L_z/2 \le z \le -H/2,$$

$$u_1 = u_0 (2z)/H,\ -H/2 \le z \le H/2 \text{ (constant gradient of } u_1\text{).}$$

Initial velocity along *y*-direction is equal 0. Initial velocity along *z*-direction has small disturbance (1% of $u_1$) inside the shear layer.

Boundary conditions are the following: periodic conditions on *x*- and *y*-directions, impermeability conditions on *z*-direction.

Figure 5 shows equiscalar surfaces of vorticity ($|\mathrm{rot}\,\mathbf{u}|$) at successive time moments for calculation with $L_z = 1$, $H = 0.2$, $L_x = 2\pi$, $L_y = \pi$.

In this case the onset of shear instability begins with the formation of large-scale vortices. This time moment corresponds with $t = 3$ (Figure 5).

Further the instability is developed on a surface of large vortices. Formed structures interact with each other and walls. Results of this processes are shown at Figure 5 at $t = 4$, $t = 5$, $t = 6$, $t = 7$ and $t = 8$.

Let is analyze the influence of length of shear layer to the evolution of the turbulence. Tree variants with different length of shear layer $L_y = 2\pi,\ \pi/2,\ \pi/8$ and with $L_x = 2\pi$, $L_z = 2\pi$, $H = 1$ were modeled. Results of these calculations are shown at Figure 6. Note that the value of the specific concentration is equal to 1 inside shear layer and 0 outside for the initial time moment.

It is shown that the evolution of the flow at the beginning has quasi-two-dimensional nature for all calculations (Figure 6, $t = 8$). Further we can see that the nature of the evolution continues to be quasi-two-dimensional longer for less value of the $L_y$. For the last calculation (Figure 6, $t = 20$) the nature of the vortex remains to be quasi-two-dimensional to the finish.

We may introduce new character number is $D/L$, where $D$ – is diameter of vortex and $L$ – is length of vortex. Law of change of velocity across shear flow determines the radius of vortex. But physical width of flow determines length of vortex. For example, in the Earth’s atmosphere width of cyclone is 10-15 km and the radius of cyclone is 200–400 km. It is shown that 2D modeling may use for the Earth’s atmosphere.

## 4. LARGE-SCALE TURBULENCE IN ACCRETION DISKS

Results in this section will be stated in compliance with papers [18]. Theoretical research of accretion disks forming around gravitating compact objects also has been conducted over many years. Recently, the problem of angular momentum transfer in accretion disks has come into prominence. Researcher’s interest in the problem is fueled by an observed link between the temperature of an accretion disk and the intensity of radiation from the compact object during mass accretion onto the object. For intensely mass accretion onto a central object, there must be processes in the disk which transfer angular momentum to its outer boundaries. [4] suggests turbulent viscosity for such a mechanism. It shows that the speed of accretion determines the heating of the accretion disk due to molecular viscosity.

There have also been attempts to suggest magnetic viscosity for this mechanism. [5] shows that even the presence of a weak magnetic field renders a hydrodynamically stable accretion disk unstable and results in turbulent flows inside the disk. This phenomenon was first considered in [6], which showed that certain distributions of the magnetic field and angular velocity result in an instability, which was termed "magnetorotational instability". Appearance of such instability brings to redistribution of angular momentum and its transfer to outer boundaries of the disk.

The general belief is that the shear-flow turbulence viscosity is local and dynamic in character and results in local emissions of heat [7]. An important problem is to explain the low temperature of the disk, which is much lower than the temperature that would account for the observed intensity of radiation at the given velocity of accretion. There are numerous works, which explore the conversion of the kinetic energy of turbulent flows not only into heat but also in other kinds of energy. This gave rise to an advection-dominated accretion [7]. Using the hydrodynamic approach, we consider an accretion disk rotating around a central gravitating compact object. Assuming that the thickness of the accretion disk is very small compared to its radius, we will be solving the problem in two-dimensional geometry. Self-gravitation of the gas will be ignored.

We shall use an ideal compressible gas whose behavior is described by the system of two-dimensional Euler gas dynamics equations in cylindrical coordinates and ideal gas equation of state.

For the boundaries of the assumed region, we set free-flow boundary conditions.

For the initial state of the accretion disk, we select the analytical solution used in [12], which is an equilibrium state obtained in [13] for a two-dimensional model.

To approximate the differential equations, we use the TVD scheme [14-17].

Figure 7 shows isolines of the vorticity  at corresponding points in time. The last point is at stage of the process, which corresponds to half-turn of the disk (the turn of the disk here stands for the time required for the disk matter in the maximum density area to rotate fully around the gravitating object).

We made a series of calculations for amplitudes ranging from 0.01 to 0.2, with disturbances introduced in one- or two-cell bands, with different numbers of local disturbances (ranging from 1 to 10 at $n = 10$ and from 1 to 20 at $n = 20$), and with various grids (from 40x130 to 320x1040). The results of the calculations show that the quality of the flow remains unchanged.

It is of some interest to observe the change in the kinetic energy of the turbulence. As mentioned above, first a disturbance of the azimuthal velocity component is introduced with amplitude $A$ ranging from 0.01 to 0.2. This corresponds to disturbance energy of 0.001% to 0.3% of the total initial kinetic energy. Figure 8 shows the change of the kinetic energy of a turbulent flow over time (the kinetic energy of a turbulence here and below is the kinetic energy of the radial movement of gas in an accretion disk). Time is measured in turns of the disk, the initial disturbance of the velocity has amplitude $A$=0.2 and $A = 0.1$. Comparing the two graphs one can notice that the maximum values are very different. Later on, when reaching the quasitationary regime, the energy values are almost the same (by the quasistationary regime we mean the state where the kinetic energy of the turbulent flow changes little over time, oscillating around an almost constant value). For the variants with $A = 0.01$ and $A = 0.05$, the values of the quasistationary regime are approximately the same as the obtained results. Therefore, we can assume that in the quasistationary regime, the kinetic energy of the turbulence is determined by the initial kinetic energy and is independent of the energy of the initial disturbance. This testifies to the physical validity of the assumption that large-scale turbulences evolve in shear flows in accretion disks and also confirms the fact that once disturbance formation has reached its peak, vortex structures do not disappear, the flow remains turbulent in character, and large structures transfer angular momentum to the outer boundaries of the disk.

Let us now analyze the change and redistribution of the angular momentum in the flow obtained in the principal calculation (Figure 7).

Figure 9 shows distribution of the angular momentum along the radius at the initial instant of time and at two full turns. Throughout the entire calculation, angular momentum is "thrown”, out of the zone where most of the matter is concentrated on both sides of the radius and is redistributed. The maximum of the angular momentum decreases compared to the initial value, and the region where the angular momentum is mainly concentrated widens. At the graph, which corresponds to two turns, the gas inside the internal boundary of this region at time zero, and particularly outside the external boundary, acquires considerable angular momentum. The above-described redistribution of angular momentum can also be observed at the graph for the difference of the angular momentums at the time when the disk material has made two full turns and at time zero.

Remember that there is no viscosity and heat conduction in the system, and the scheme viscosity is small. The constant entropy means that the °ow is practically adiabatic. Consequently, turbulent viscosity is small and cannot act as a mechanism for the redistribution of the angular momentum. Thus, the redistribution of the angular momentum observed in the calculations is related to the formation of large structures. It is these very structures that transfer the angular momentum.

## 5. CONCLUSION

In free shear flow the birth of turbulence connects with large scales. Zhoukovskij force plays the important role in the evolution of turbulence. The ratio $L_y/L_x$ determinate 2D assumption. If $L_y/L_x \ll 1$ then 2D model can be used. The thin accretion disk concerned in our calculation satisfy to this condition.

The disk configuration studied in this paper, with density at the boundaries by several orders smaller than the density in the center, was chosen so as to exclude any impact of boundary conditions.

The calculations carried out enable us to make several conclusions as to the development of small disturbances. Small disturbances, introduced in a relatively small region of an accretion disk in the state of stability, develop into large structures and spread over a considerable portion of the assumed region. The resulting flow is turbulent in character.

These large structures play a major role in redistribution of the total angular momentum in accretion disks without any noticeable heating of the disk material. However, there is no evidence in the calculations of any reductions in the total angular momentum of the disk material, nor of any ejection of material.

We thank Joint Supercomputer Center for providing of power parallel supercomputer resources for numerical simulation.

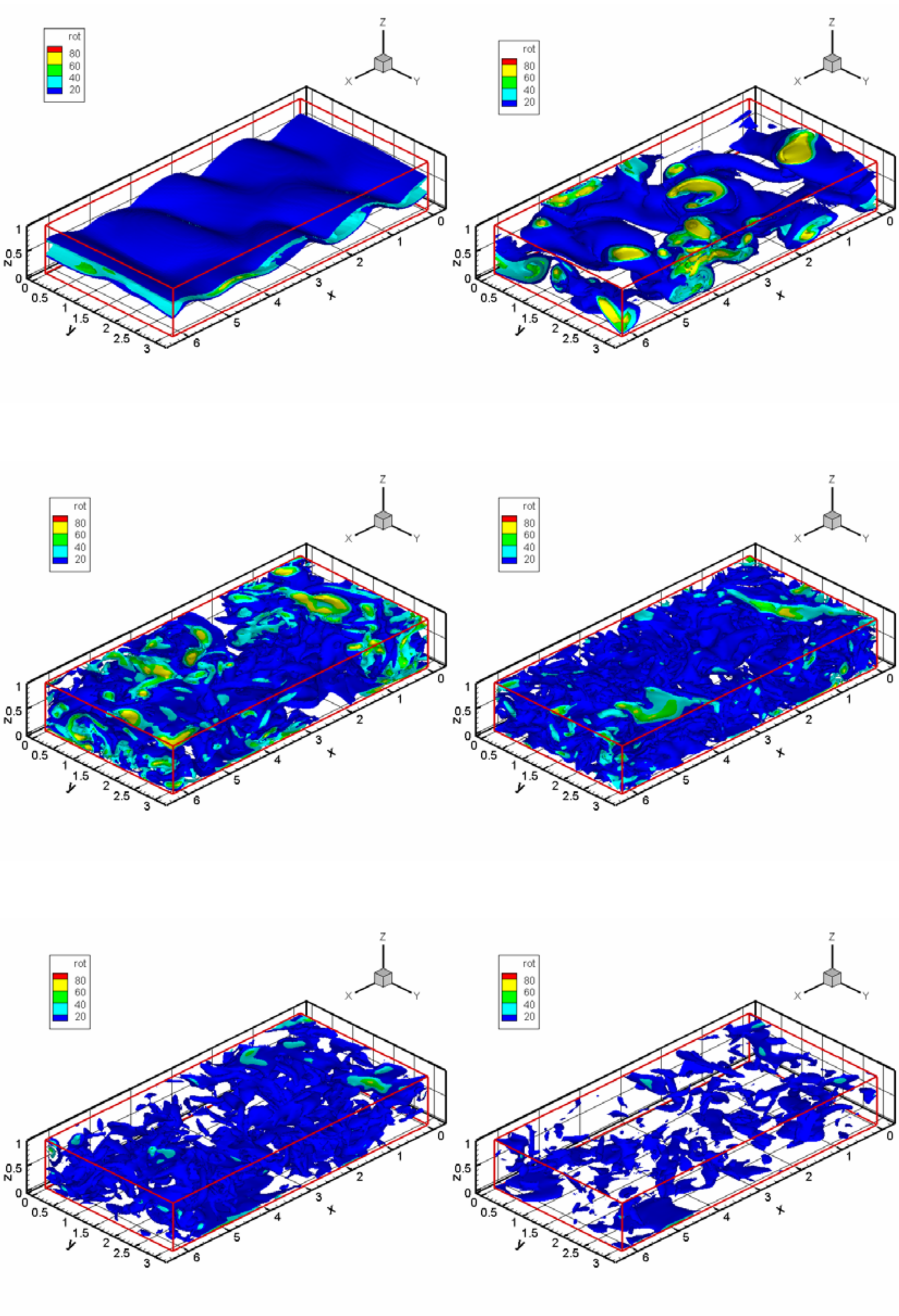


Figure 5. Equiscalar surfaces of the vorticity at successive time.

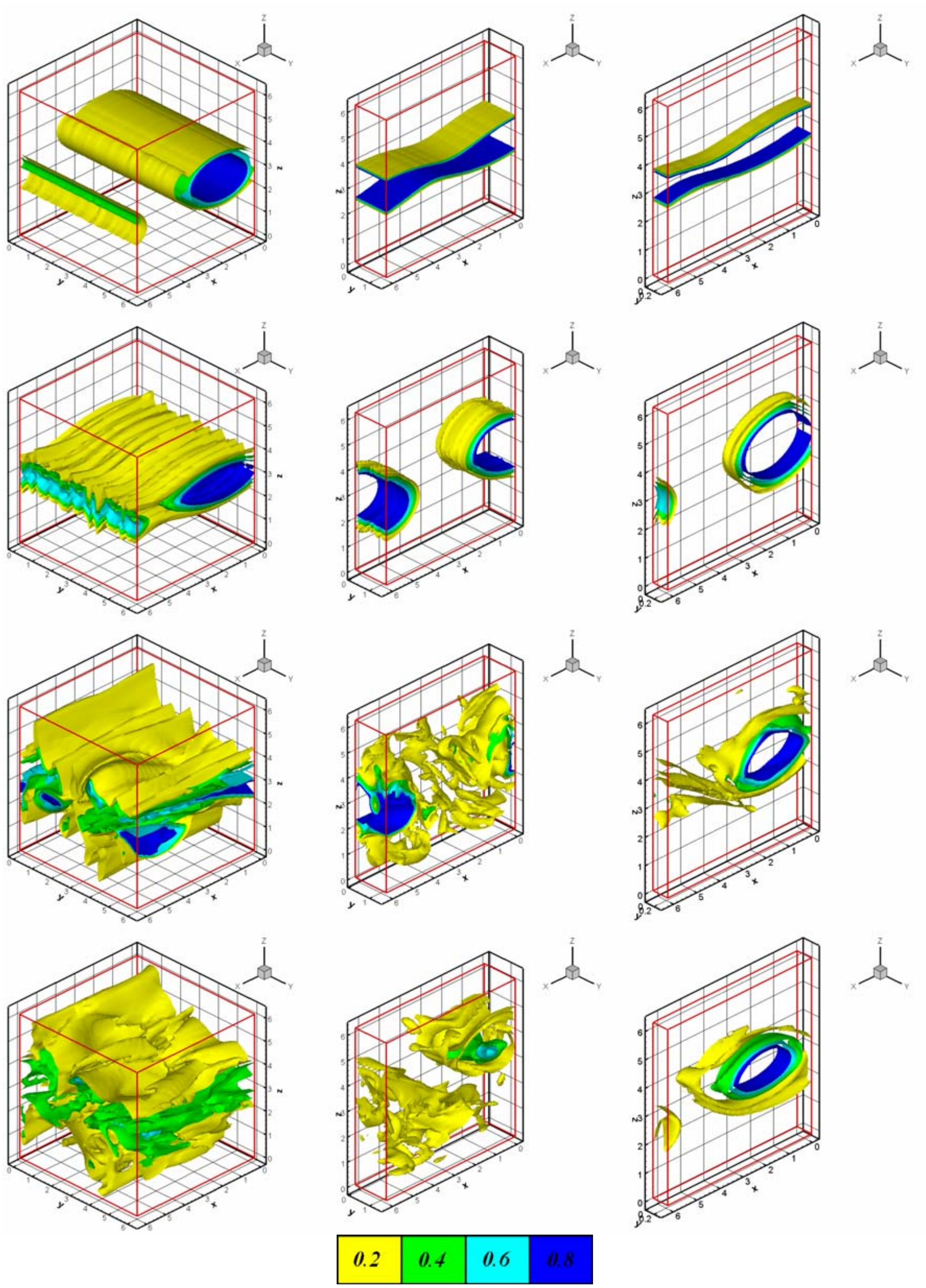


Figure 6. Equiscalar surfaces of the specific concentration. Time moments (from top to bottom) - 8, 12, 16, 20. Length of the shear layer $L_y = 2\pi,\ \pi/2,\ \pi/8$ .

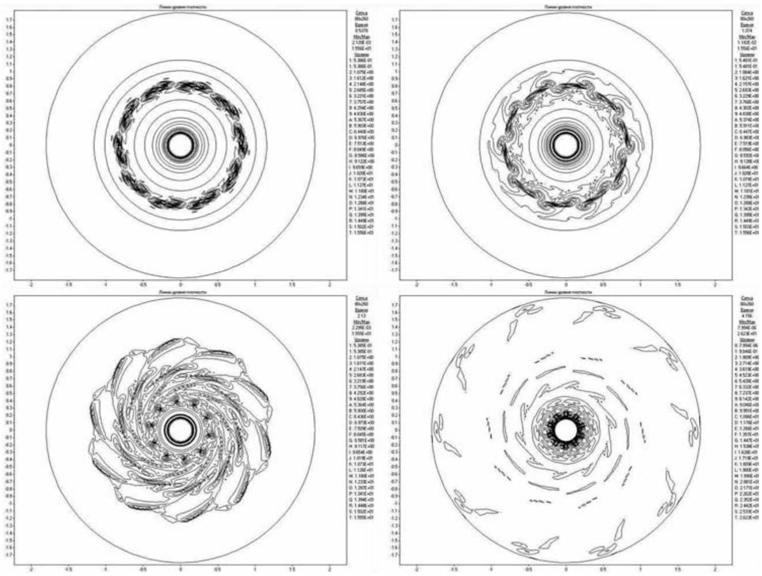

Figure 7. Isolines of the vorticity.

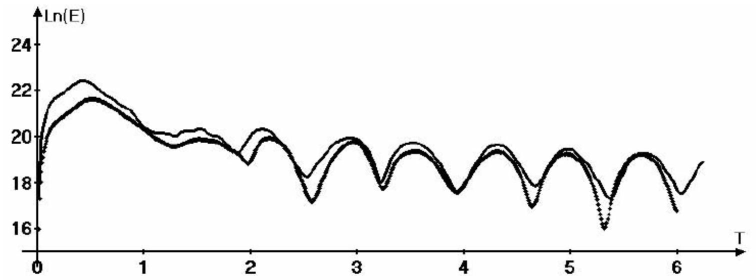


Figure 8. Change of the kinetic energy of a turbulence over time (in disk turns) at initial disturbances of velocity with $A$ = 0.2 (solid line) and $A$ = 0.1 (dots).

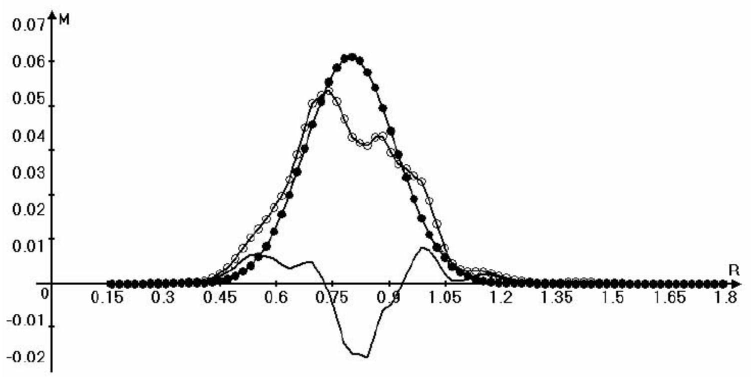


Figure 9. Redistribution of the angular momentum along the radius ($\varphi = 0$) at times $t_1$ = 0 (black dots line) and $t_2$ = 2 turns (white dots line) and the difference of the angular momentums $M(t_2) - M(t_1)$ (solid line).

## 6. REFERENCES


[1] O.M. Belotserkovskii, Zh. Vychisl. Mat. Mat. Fiz., 25, 1856 (1985) [in Russian].

[2] O.M. Belotserkovskii, *Turbulence and Instabilities*., Lewinston, Edwin Mellen Press, (2000).

[3] O.M. Belotserkovskii, A.M. Oparin and V.M. Chechetkin, *Turbulence: New Approaches*, Moscow, Nauka, (2002).

[4] N.I. Shakura, Sov. Astron., 16, 756 (1973) [in Russian].

[5] S. Balbus and J. Hawley, Astrophys. J., 376, 214 (1991).

[6] Ye.P. Velikhov, Zh. Exp. Teor. Fiz., 36, 1399 (1959).

[7] R. Narayan and I. Yi, Astrophys. J., 452, 710 (1995).

[8] J.G. Lominadze, G.D. Chagelishvili and R.G. Chanishvili, Sov. Astr. Letters, 14, 364 (1988).

[9] G.D. Chagelishvili, J.P. Zahn, A.G. Tevzadze and J.G. Lominadze, Astron. and Astrophys., 402, 401 (1988).

[10] A.G. Tevzadze, G.D. Chagelishvili, J.P. Zahn et. al. Astron. and Astrophys., 408, 779 (2003).

[11] G. Bodo, G.D. Chagelishvili, G. Murante et. al., Astron. and Astrophys., 437, 9 (2005).

[12] M.V. Abakumov, S.I. Mukhin, Yu.P. Popov and V.M. Chechetkin, Astron. and Astrophys., 47, 11 (2003).

[13] M.V. Abakumov, S.I. Mukhin, Yu.P. Popov and V.M. Chechetkin, Astron. Rep., 40, 366 (1996).

[14] P.L. Roe, Ann. Rev. Fluid Mech., 18, 337 (1986).

[15] S. Osher and F. Solomon, Math. Comput., 38, 339 (1982).

[16] S. Chakravarthy and S. Osher AIAA Pap., 85-0363, (1985).

[17] B. Einfeld, SIAM J. Numer. Anal., 25, 294 (1988).

[18] Ye.P. Velikhov, A.Yu. Lugovsky, S.I. Mukhin, Yu.P. Popov, and V.M. Chechetkin, *The Impact of Large-Scale Turbulence on the Redistribution of Angular Momentum in Stellar Accretion Disks*, *Astronomy Reports*, Vol. 51, No. 2, 154–160 (2007).