

## TO THE RESONANT SOUND-ABSORBING STRUCTURES CALCULATION BASED ON IMPEDANCE METHOD

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**Abstract.** In this paper a several types of resonant sound-absorbing structures (one-, two- and three-layered, with rectangle and trapezoid vertical profile of partial resonators) computationally investigated. Investigation based on analytical calculations of linear impedance and absorption coefficient. A good agreement with physical concept and other authors data was shown. Additional parametrical optimization (unused of other authors) allows to achieve more SAS efficiency. Obtained results can be useful in SAS design and noise prediction for modern aircraft with high-by-pass engines.

### 1. INTRODUCTION

Today one of basic aviation problems is an aircraft engine noise reduction problem. For their solution a special sound-absorbing structures (SAS) applied. Usually it installed in inlet duct and external engine duct. For fan noise reduction in forward engine hemisphere SAS installes at internal inlet surface. In case of backward hemisphere SAS installes to external engine countour surface.

On of effective SAS types is a resonant SAS. Its contain a set of resonators tuned to different frequencies, presenting in engine spectrum. In practice this spectrum include a wide frequency range. Consequently a SAS with wide absorption line needed. This problem can be solved by taking into account of acoustic wave incidence angle and, generally, by including dissipation into system. Last goal achieved by different methods and one of more using is an application of perforated plates covering resonator cavity. Also a non-rectangle (triangle, trapezoidal etc.) vertical profile of resonator cavity proposed for absorption line broadening. For tuning to different frequencies leading to absorption line broadening a multilayered SAS used.

Engineering practice require an exact (practically acceptable) acoustical calculation method for SAS design. Note that standard method<sup>1,2</sup> didn't take to account a number of holes in perforated plate, located under single resonator. In this paper an absorption coefficient calculations realized taking into account a number of holes under single resonator cavity for a resonant single- and multilayered SAS with perforated plates with rectangle and trapezoidal resonator cavity profiles. Obtained results have more physical

sense and are in good agreement with experimental data and calculations of other authors<sup>3,4</sup>.

## 2. SAS CALCULATION METHOD

### 2.1. Standard calculation method.

A basic SAS scheme represented at fig. 1.

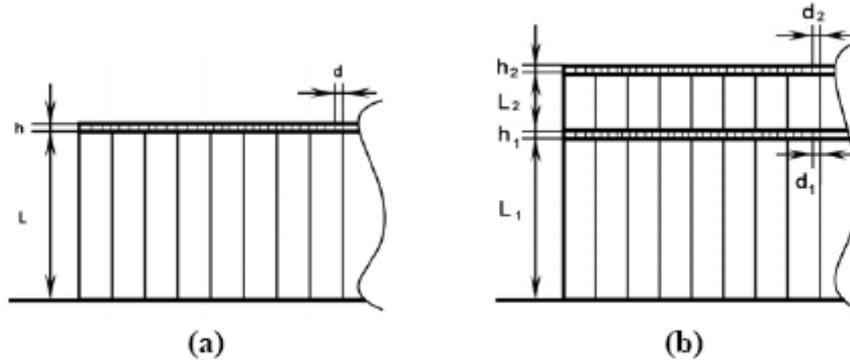


Figure 1. SAS scheme for one- (a) and two-layered (b) construction with rectangle resonator vertical profile.

In a standard calculation method a full sound absorption condition using for basic calculation formulas for impedance and absorption coefficient obtaining<sup>1</sup>:

$$kM = ctg(kL),$$

$$M = \frac{h}{n} \left[ 1 + \frac{\pi d}{4 \cdot h \cdot F(\sqrt{n})} \right], \quad (1)$$

where  $M$  is an attached mass, i.e., effective mass of air in perforate panel holes,  $L$  is a resonator height,  $d$  is a diameter of holes in perforated panel,  $h$  is a perforated plate thickness,  $n$  is perforation percentage (ratio between total area of holes under single resonator and total area of plate part covering this resonator) and  $F(\sqrt{n})$  is a Fock's function. Last function can be expressed<sup>1</sup> as a series by  $\sqrt{n}$ . V.S. Nesterov in 1941 propose an empirical formula for Fock's function (these formula have been used later by Müller<sup>5</sup>), which give a better approximation<sup>1</sup> and used in this work:

$$F(\sqrt{n}) = \left( 1 - 1.47 \cdot \sqrt{n} + 0.47 \cdot (\sqrt{n})^3 \right)^{-1}. \quad (2)$$

In this method supposed that only one hole in perforated panel located under single resonator and several holes equates with one hole of more diameter. In fact it mean that there is no interaction between air in different holes.

For one-layered SAS with rectangle resonator vertical profile an input linear impedance expressed by follow formula:

$$Z_{\text{onelayer}} = 1 + i \cdot \left( \frac{2\pi}{c} f \cdot \frac{h}{n} \left( 1 + \frac{\pi d}{4h \cdot F(\sqrt{n})} \right) - \text{ctg} \left( \frac{2\pi}{c} f \cdot L \right) \right), \quad (3)$$

where  $c$  is a sound velocity and  $f$  is a sound frequency. Formula for  $l$ -th layer impedance in multi-layered SAS can be expressed as:

$$Z_l = 1 + i \cdot \left\{ \frac{2\pi}{c} f \cdot \frac{h_l}{n_l} \left[ 1 + \frac{\pi d_l}{4h_l \cdot F(\sqrt{n_l})} \right] - \text{ctg} \left( \frac{2\pi}{c} f \cdot L_l \right) \right\}. \quad (4)$$

And formulas for two- (fig. 1b) and three-layered SAS written below:

$$Z_{\text{two layers}} = Z_1 + \frac{1}{\sin^2 \left( \frac{2\pi}{c} f \cdot L_1 \right) \cdot Z_2},$$

$$Z_{\text{three layers}} = Z_1 + \frac{1}{\sin^2 \left( \frac{2\pi}{c} f \cdot L_1 \right) \cdot \left[ Z_2 + \frac{1}{\sin^2 \left( \frac{2\pi}{c} f \cdot L_2 \right) \cdot Z_3} \right]}, \quad (5)$$

Absorption coefficient can be expressed through impedance:

$$t = 1 - \left| \frac{Z - 1}{Z + 1} \right|^2. \quad (6)$$

A using of standard method for calculation one can obtain that for one- and two-layered SAS with rectangle vertical resonator profile an absorption coefficient increase and frequency range of effective absorption widens with perforation percentage increasing. This result is not correspond to experimental data<sup>4</sup> and general physical conceptions. Consequently, it's needed to specify standard method.

## 2.2. A sound wave incidence angle taking into account.

An engine fan in working process (fig. 2) “modulate” an air flow and creates acoustic radiation of frequency  $N \cdot f_F$ , where  $N$  is a number of fan blades and  $f_F$  is a fan shaft rotation frequency.

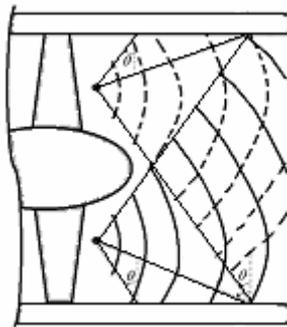


Figure 2. A wave propagation from fan blade ( $\theta$ - sound wave incidence angle).

This radiation is not a plane or spherical waves and have a complicated form because of blade form is very complicated and their size is congruent with inlet size. So, it's impossible to exactly describe a real wavefront propagation from fan blade but it's possible to choose an acceptable physical model: two spherical wave sources on each side of fan axis (fig. 2, sources shown by black points).

In this case formula for absorption coefficient transformates to follow expression:

$$t = 1 - \left| \frac{Z \cdot \cos \theta - 1}{Z \cdot \cos \theta + 1} \right|^2. \quad (7)$$

### 2.3. A specified calculation method: a number of holes taking into account

As written above standard SAS calculation method lead to non-physical results. One of main reason of such results may be a failure to take account of number of holes located under single resonator. In case of real construction usually a several holes in perforated panel located under single resonator. Each hole represent a separate resonator (called further elementary resonator) and total impedance of single resonator defines as a parallel connection of such elementary resonators.

A number of holes ( $N$ ) can be calculated using perforation percentage, resonator size  $a$  (in plane of perforated panel) and distance between holes  $b$ :

$$N = \frac{a^2}{b^2}. \quad (8)$$

An expression for total impedance then can be expressed by formula:

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_N}, \quad (9)$$

where elementary resonator impedances  $Z_1, Z_2 \dots Z_N$  calculates by standard method<sup>1</sup>.

Then absorption coefficient can be calculated using expression (6) or (7).

In this approach a contradiction described above disappear: sound absorption efficiency increase with perforation percentage decreasing (in "working" range, it will be demonstrated below that in general case this dependence have an extremal character).

## 3. COMPUTATIONAL RESULTS

### 3.1. An incidence angle influence estimation on SAS efficiency

In this chapter a sound absorption angular dependence considered for two-layered SAS with rectangle vertical resonator profile. A SAS parameters using for calculation represented in table 1.

Parameter	1 <sup>st</sup> layer (closer to flow)	2 <sup>nd</sup> layer
$L$ , mm	22	8
$h$ , mm	1,2	1,2
$R$	1	1
$d$ , mm	2	2
$n$ , %	8	5

Table 1. SAS parameters.

An absorption coefficient frequency characteristics for different incidence angle represented at figure 3.

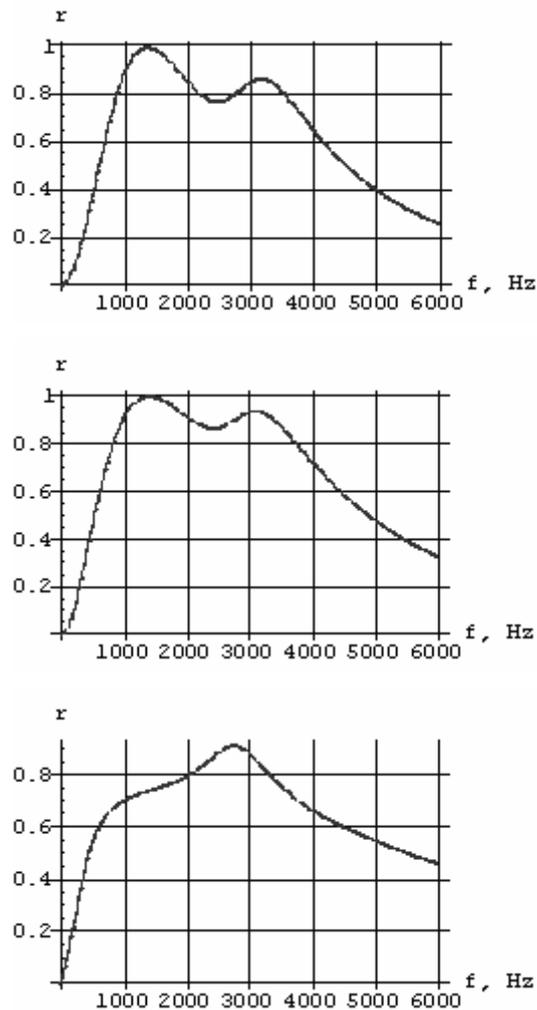


Figure 3. An absorption coefficient frequency characteristics for different incidence angles ( $0^\circ$ ,  $45^\circ$  and  $75^\circ$  for upper, middle and lower characteristic correspondingly).

From figure 3 one can obtain a collapse presence at 2.5 kHz which magnitude varies from 0.75 to 0.9. It's also clear that for incidence angle equal to  $75^\circ$  a peak of magnitude 1 appears at 2.7 kHz. Generally, one can see that absorption coefficient is not strongly dependent from incidence angle and their influence is not determinant.

### 3.2. SAS calculation using specified method

Parameters used for calculations by specified method chose based on data about real modern SAS and shown in table 2.

Parameter	Value(-s)
$c$ , m/s	340
$d$ , mm	2
$h$ , mm	1,2
$a$ , mm	4,5; 8; 20
$n$ , mm	1; 3; 5; 8; 10; 15
$f$ , Hz	0÷5000
$L$ , mm	8; 15; 22

Table 2. Parameters for SAS calculation by specified method.

An absorption coefficient frequency characteristics for one-, two- and three-layered SAS with rectangle vertical resonator profile represented at figure 4.

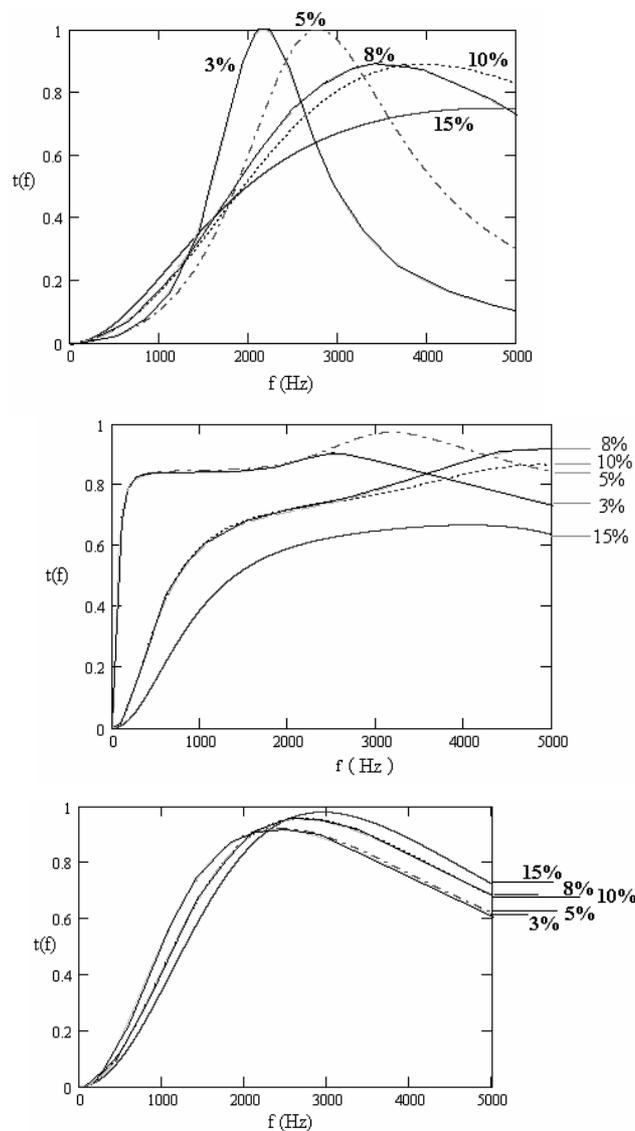


Figure 4. An absorption coefficient frequency characteristics for different SAS with rectangle vertical resonator profile. Upper characteristic – one-layered SAS ( $L = 8$  mm;  $n = 3\%$ ,  $5\%$ ,  $8\%$ ,  $10\%$ ,  $15\%$ ;  $a = 8$  mm), middle – two-layered SAS ( $L_1 = 22$  mm;  $L_2 = 8$  mm;  $n_1 = 15\%$ ;  $n_2 = 3\%$ ,  $5\%$ ,  $8\%$ ,  $10\%$ ,  $15\%$ ;  $a = 8$  mm), lower – three-layered SAS ( $L_1 = 22$  mm;  $L_2 = 8$  mm;  $L_3 = 5$  mm;  $n_1 = 10\%$ ;  $n_2 = 1\%$ ,  $3\%$ ,  $5\%$ ,  $8\%$ ,  $10\%$ ,  $15\%$ ;  $n_3 = 10\%$ ;  $a = 8$  mm).

A scheme of one-layered SAS with trapezoid vertical resonator profile represented at figure 5 (this SAS have a wide “neck”, also a SAS with narrow “neck” used – converged upward) and corresponding absorption coefficient frequency characteristics shown at figure 6.

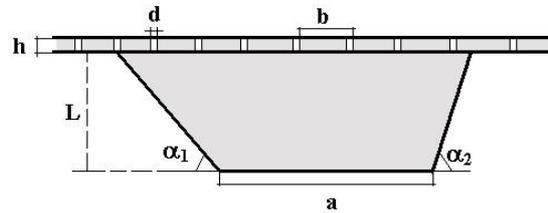


Figure 5. A scheme of one-layered SAS with trapezoid vertical resonator profile.

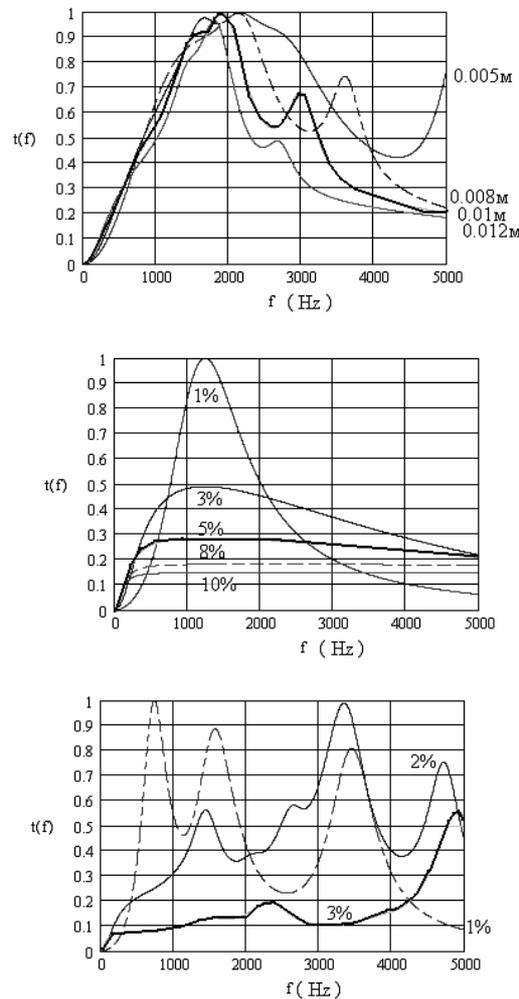


Figure 6. An absorption coefficient frequency characteristics for one-layered SAS with trapezoid vertical resonator profile. Upper characteristic – SAS with wide “neck” ( $L = 22$  mm;  $\alpha_1 = \alpha_2 = 65^\circ$ ;  $n = 2\%$ ;  $a = 5$  mm, 8 mm, 10 mm, 12 mm), middle – SAS with narrow “neck” ( $L = 22$  mm;  $\alpha_1 = \alpha_2 = 55^\circ$ ;  $n = 1\%$ , 3%, 5%, 8%, 10%;  $a = 60$  mm), lower – parallel connection of SAS with wide and narrow “neck” ( $L = 22$  mm;  $\alpha_1 = \alpha_2 = 55^\circ$ ;  $n = 1\%$ , 2%, 3%;  $a = 60$  mm (narrow), 8 mm (wide)).

Obtained results allow to conclude following. A two-layered SAS have more broadbandness (more than 0.8) in frequency range from 500 to 5000 Hz for some layers heights and perforation percentages (fig. 4). Three-layered SAS (as seen from

computational investigation) have less efficiency than two-layered SAS (fig. 4). A tendency of sound absorption efficiency increasing with perforation percentage decreasing have seen from fig. 4 (middle graph) which agree with data of other authors<sup>3,4</sup>. This fact have a clear physical sense: strongly perforated panels are almost transparent for air flow and can't promote to flow energy effective absorbtion (although can additionally turbulize an air flow). On the other hand very slightly perforated panels can't to dissipate flow energy, although its can supply a high absorption coefficient in narrow frequency band (it's also clearly seen at upper graph at fig. 4 for one-layered SAS). So, a dependence of sound absorption effeciency from perforation percentage have an extremal character, although in "working range" can be described by inverse dependence. Moreover, a resonator size appear significant. It can be obtained in both of standard and specified calculation methods than in case of small internal layer height a two-layer SAS with rectangle vertical resonator profile work as one-layered SAS of larger height (absorption peak shifts to low-frequency spectral band)<sup>6</sup>. This fact correspond to results of A.F. Sobolev<sup>3</sup> but in his work only two-layered SAS with equal perforation percentages of layers studied. And low efficiency of such SAS used as basis for conclusion about inexpediency of SAS with perforated panels using in modern noise reduction systems<sup>3</sup>. In this work a SAS with non-equal perforation percentages of layers studied. From obtained calculation data one can see that optimal choise of perforation percentages for both of two layers allow to improve SAS characteristics and significantly increase their efficiency. Note that from physical conceptions and computational data follows that external layer must have more perforation percentage than internal: it supply a better energy dissipation in resonator. Furthermore computational results for one-layered SAS with trapezoidal vertical resonator profile (fig. 5-6) are evidence of radiation mode existence for SAS with wide "neck". This mode is absent for SAS with narrow "neck" and for parallel connection of SAS with wide and narrow "necks". Generally, SAS with trapezoid vertical resonator profile extremely sensitive to values and ratio of angles at trapezium base.

Figure 7 demonstrate a comparison of computational results obtained in this paper with experimental data<sup>3</sup>. It's shown that calculation give acceptable results.

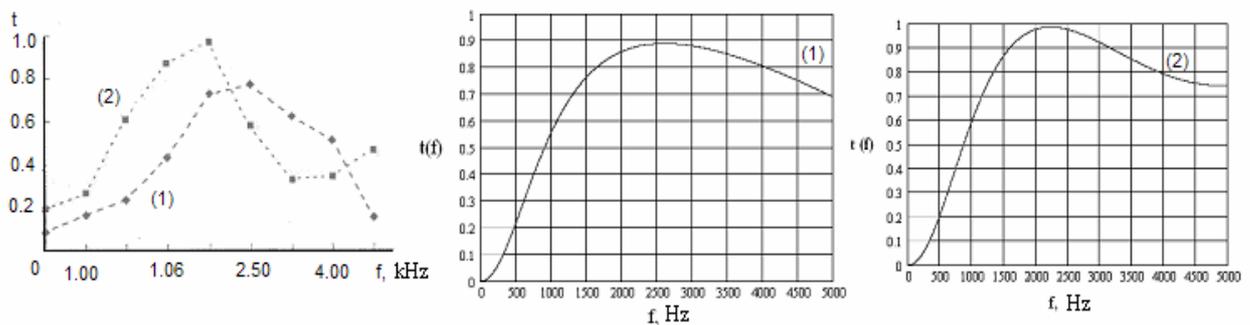


Figure 7. A comparison of computational and experimental results. Left graph – experimental results<sup>3</sup> (1 – one-layered SAS,  $L = 20$  mm; 2 – two-layered SAS,  $L_1 = 14$  mm and  $L_2 = 6$  mm), middle graph – computational results for one-layered SAS ( $L = 20$  mm;  $n = 10\%$ ;  $h = 0.8$  mm;  $d = 1.5$  mm), right graph – computational results for two-layered SAS ( $L_1 = 14$  mm;  $L_2 = 6$  mm;  $n_1 = n_2 = 10\%$ ;  $h = 0.8$  mm;  $d = 1.5$  mm). For two-layered SAS index 1<sup>st</sup> layer is a layer which close to flow.

Generally, a good agreement with basic physical conceptions and experimantal data allow to hope to accuracy and adequacy of specified SAS calculation method and chosen approach.

Note that all of these calculations carried out for two-dimensional geometry of SAS. For some modern SAS types as chevron SAS it's not correct and three-dimensional case must be considered.

#### 4. CONCLUSION

So, in this work a specified calculation method for SAS proposed and verified. It's shown that computational results are in good agree with other author's data and with basic physical conceptions. A basic SAS characteristics behaviour described correctly by proposed method. For two-layered SAS with rectangle vertical resonator profile calculation data correspond quantitatively and qualitatively to experimental and computational data<sup>3,4</sup>. Furthermore, it's shown that two-level SAS with non-equal perforation percentage of both of layers (non-studied in work<sup>3</sup>) supply a more efficiency (more broadbandness and more value of absorption coefficient). This fact allow to increase SAS efficiency by optimization of layers perforation percentage and ratio of perforation percentages for two layers<sup>6</sup>. For one-layered SAS with trapezoid vertical resonator profile it's shown that in case of wide "neck" a radiation mode can appear. Also for such construction obtained a significant sensitivity to angles at trapezium base.

Obtained results can be useful for aircraft noise reduction systems design and testing.

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