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GASDYNAMIC DESIGN OF SHAPED NOZZLES FOR SUPERSONIC WIND TUNNELS, ALLOWING FOR VISCOSITY

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Abstract. Supersonic shaped nozzles are used in gasdynamic test facilities for providing uniform gas flow. The article explains the method which takes into account the effect of viscosity on the shape of a supersonic axisymmetric nozzle designed to ensure a specified Mach number from 5 to 10; the method relies on the boundary layer theory. Presented are generalized equations for the reduced displacement thickness δ^* of laminar and turbulent boundary layers, which is regarded as a viscosity-related correction to the ideal (isentropic) nozzle shape.

The procedure of designing the changeable transonic sections for the basic nozzle is described; the sections make it possible to ensure the exit flow speed field to be uniform over a certain Mach number range. The theoretical results are validated by experiments dealing with a basic M = 10 nozzle with the sections designed to provide M = 4–9.

The changeable sections were also used to widen the operational Mach number ranges of nozzles previously installed of wind tunnels. For example, the basic M = 10 nozzle had been complemented with three changeable sections providing M_i = 7.0, 7.5 and 8.2. Test results demonstrate satisfactory convergence of theoretical and experimental flow fields in the wind tunnel test section.

The article describes a more comprehensive method for solving the inverse problem of designing the supersonic nozzle with allowance for viscosity so that flow in the inviscid core is uniform at a specified Mach number. The method is also applicable in the case of turbulent flow in the boundary layer. The method is exemplified with design of the shape, $y_w(x)$, of a M = 18 nozzle for a cryogenic vacuum wind tunnel (with a laminar gas flow). Theoretical and experimental Mach number fields at the nozzle exit plane converge satisfactorily.

1. Design of the supersonic part of an isentropic surface shape $y(x)$ of an axisymmetric nozzle relies on the hypothesis of a sonic line at the critical plane being straight and is based on the method of characteristic curves (i.e., the method of determining flow lines by Mach characteristics) for a prescribed axial distribution of Mach number, $M_0(x)$, over the flow acceleration part. The distribution M_0 is defined as a piecewise smooth function for five x-axis intervals: a parabola, a straight line, lines implying a radial gas flow over two intervals, and a second parabola. The function M_0 for the last interval ensures the nozzle curvature not to be discontinuous in the domain where the accelerating part of the nozzle is attached to the smoothing part thereof.

2. The influence of viscosity on the flow is taken into account by increasing the isentropic surface radius by the boundary layer displacement thickness δ^*; refer to [1, 2]. The present

authors had summarized their results of experimental study of air flow in wind tunnel nozzles with turbulent boundary layers and added the data from other researchers to propose a function $\left(\delta_1^*/x\right)\mathrm{Re}_x^{0.2}$ of Mach number for the range from 1.0 through to 10; here, δ_1^* is the displacement thickness corresponding to the temperature factor $\overline{T}_w = T_w/T_0 = 1$, where T_0 is the stagnation temperature and T_w is the wall temperature.

In tests without preheating the air in the stilling chamber (when $\overline{T}_w = 1$) the δ_1^* value is determined in immediately the test; in the case of $\overline{T}_w < 1$ the value is calculated by relevant formulae. The latter use the dependence of δ^*/δ_1^* on the temperature factor: $\delta^*/\delta_1^* = 0.65 + 0.35\overline{T}_w$, as provided by approximation of available test data. The dependence of the reduced displacement thickness on Mach number at $\overline{T}_w = 1$ is as follows:

$$\frac{\delta_1^*}{x}\mathrm{Re}_x^{0.2} = f_t(\mathrm{M}),$$

where $f_t(\mathrm{M}) = 0.046 + 0.014(\mathrm{M}-1) + 0.0047(\mathrm{M}-1)^2 - 0.00015(\mathrm{M}-1)^3$.

For other $\overline{T}_w$ values the reduced displacement thickness is defined with,

$$\left(\frac{\delta^*}{x}\right)\mathrm{Re}_x^{0.2} = \left(0.65 + 0.35\overline{T}_w\right) f_t(\mathrm{M}).$$

In case the boundary layer is laminar ($\mathrm{Re}_x < 10^6$) the δ^* value is computed by numerically solving equations for the laminar boundary layer. However, the preliminary aerodynamic design would be based on simple expression of the reduced displacement thickness of the laminar boundary layer as a function of M and the temperature factor. Expressions of such kind had been derived (1) for the laminar boundary layer over a plate in a flow with M = 0–10 and (2) for a shaped nozzle designed to be operated at $\mathrm{M}_p = 10$; the air was assumed to have stagnation temperature T_0 = 300 and 1000 K. The expressions include

– dependence of $\left(\delta_1^*/x\right)\mathrm{Re}_x^{0.5}$ on M at $\overline{T}_w = 1.0$ and

– dependence of δ^*/δ_1^* on $\overline{T}_w$ at various Mach number values.

In this case the dependence of the reduced displacement thickness on M at the inviscid nozzle surface at $\overline{T}_w = 1.0$ may be approximated as follows:

$$\left(\frac{\delta_1^*}{x}\right)\mathrm{Re}_x^{0.5} = f_l(\mathrm{M}), \quad f_l(\mathrm{M}) = 1 + (\mathrm{M}-1) + 0.037(\mathrm{M}-1)^2 + 0.0343(\mathrm{M}-1)^3.$$

The function $\delta^*/\delta_1^* = f\left(\overline{T}_w, \mathrm{M}\right)$ is linear with respect to $\overline{T}_w$ and quadratic with respect to M. These relations evaluate the displacement thickness in a nozzle for $\mathrm{M}_p \le 10$, while not requiring the laminar boundary layer flow to be finely calculated by time-consuming programs.

3. Shaped nozzles with changeable critical sections [2, 3] are of considerable practical interest. As in the case of conical nozzles with changeable sections, the shaped nozzles offer varying the exit Mach number and a suitable uniformity of the gas speed field at the

exit plane over a certain Mach number range. Designing the shaped nozzle is generating the shape of the introductory part which ensures that the flow speed field at the exit of the fixed nozzle part is almost uniform at $M = M_i$ (let us assume that $M_i < M_p$). This approach changes the critical section radius r_{*i}, thus changing the shape of the changeable supersonic section which smoothly mates with the output part. To implement the approach with the method of characteristic curves, one should solve the main problem of inviscid flow for the nozzle portion from the exit plane to the critical plane, with the solution starting from the rectilinear characteristic line that corresponds to $M = M_i$.

The calculation provides parameters of flow in the smoothing part, including the distribution $M_{0i}(x)$ along the axis. The calculation assumes a conditional isentropic surface $y(x)$ whose coordinates are obtained by subtracting the displacement thickness $\delta_i^*(x)$ from coordinates of the basic nozzle, $y_w(x)$. The stage 2 in the algorithm is to smoothly mate (1) the $M_{0i}(x)$ function found for the smoothing part and (2) the $M_{0i}(x)$ function specified for the acceleration part, and thereafter to solve the inverse problem on shaping the supersonic changeable in correspondence with the overall distribution $M_{0i}(x)$.

The algorithm has been employed to complement the $M_p = 10$ basic nozzle with changeable transonic parts which provide the exit flow with $M_i = 4$–9. The exit flow Reynolds number Re_x was $4{\cdot}10^6$.

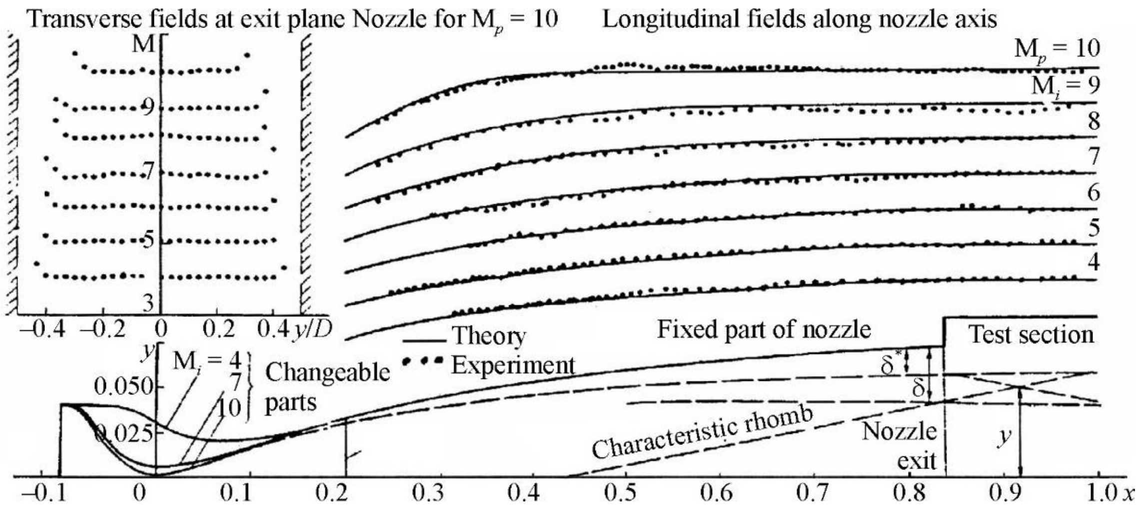

Figure 1. Flow field in nozzle with changeable parts – M from 4 to 10

Figure 1 compares theoretical results and test data. It is seen that there in the exit plane flow are very insignificant gradients along the axis, and M_i is very close to specifications. Theoretical and experimental $M_0(x)$ functions may be said to converge satisfactorily. The top part of the Figure represents transverse distributions of M over the exit plane. For all the Mach numbers the nozzle exit flow features a uniform core speed field for both the transverse and longitudinal directions (the nonuniformity extent, ΔM/M, does not exceed one per cent, – in line with this for the longitudinal direction).

The design method was also used to widen the operational Mach number ranges of nozzles previously installed of wind tunnels – in particular, the nozzle designed to provide $M_i = 10$ in the TsAGI T-117 hypersonic wind tunnel [3]. The nozzle had been complemented with new critical parts ensuring the exit plane section flow to be almost uniform at $M_i = 7.0$, 7.5 and 8.25.

Figure 2 compares theoretical and experimental data for the M = 7.5 changeable part in which the air has the total pressure $p_0 = 1.5$ MPa and the stagnation temperature $T_0 = 700$ K. These compare satisfactorily. Figure 3 depicts the values of p_0'/p_0 measured at several stations in the test section; here, p_0' is pressure downstream of the bow shock wave (upstream of the Pitot pressure tube).

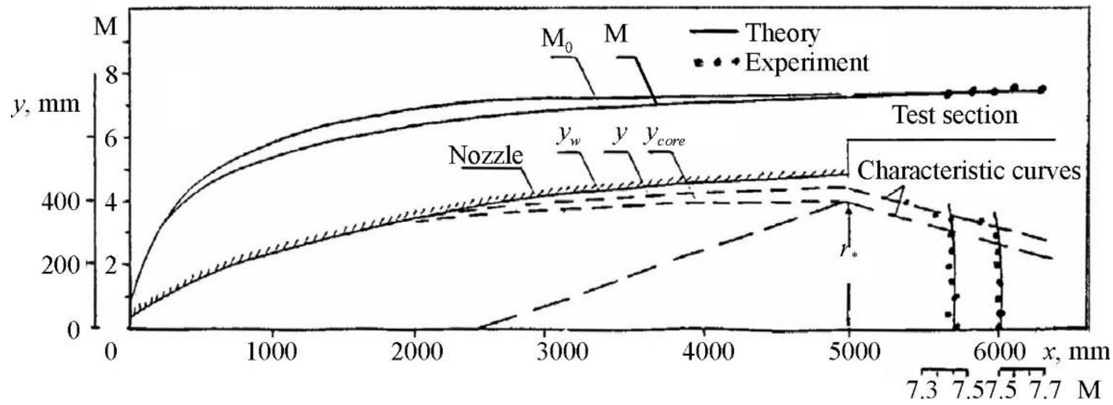

Figure 2. Comparison of theoretical and experimental flows ($M_i = 7.5$, $p_0 = 1.5$ MPa, $T_0 = 700$ K)

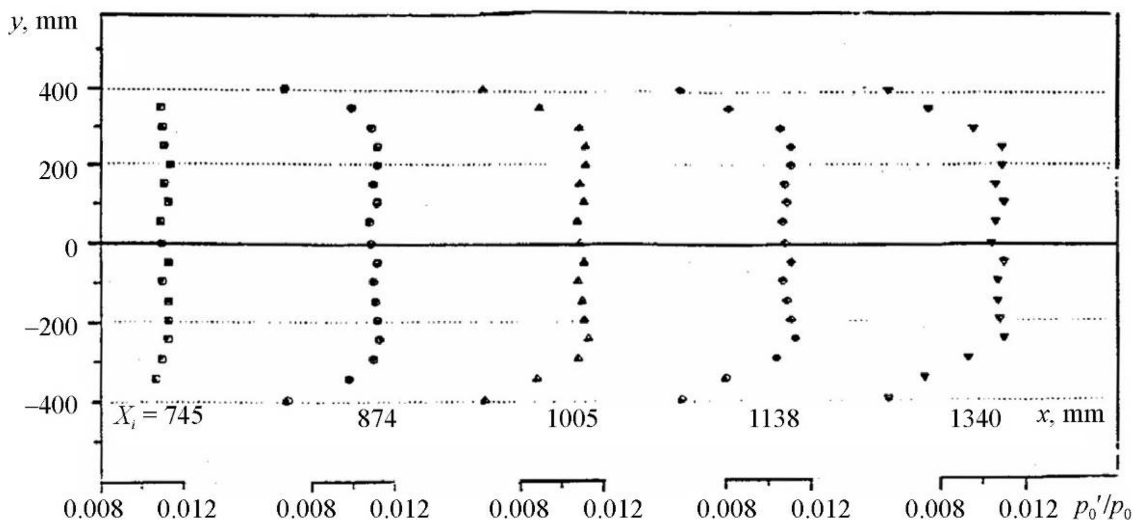

Figure 3. Distributions of p_0'/p_0 in wind tunnel stations ($M_i = 7.5$, $p_0 = 1.5$ MPa, $T_0 = 700$ K)

4. In a hypersonic nozzle the boundary layer thickness is comparable with the nozzle radius (i.e., $\delta \sim y_w$); therefore, applicability of the boundary layer theory was in doubt. In this case the inverse problem for the viscous flow should (as in [4, 5]) be solved so as to implement the specification for the Mach number distribution along the inviscid flow core in the nozzle with the surface $y_w(x)$ (to be determined): the distribution must coincide with the $M_0(x)$ function corresponding to inviscid flow in the nozzle with the surface $y(x)$. This condition means that characteristic curves of the two flows must coincide within the inviscid core.

The surface $y_w(x)$ would be determined by successive approximations [6]. Initially, the function $\Delta^{(1)}(x) = y_w(x) - y(x)$ is assumed to be equal to the longitudinal distribution of the nozzle wall boundary layer displacement thickness $\delta^*(x)$ (on the basis of numerical solutions for laminar flow and of empirical relations for turbulent flow). This provides a rough solution for the problem:

$$y_w^{(1)}(x) = y(x) + \Delta^{(1)}(x) = y(x) + \delta^*(x).$$

Thereafter, simplified Navier–Stokes equations for the viscous flow in the nozzle with the surface $y_w^{(1)}(x)$ are solved under the initial assumptions (uniform flow speed field; no boundary layer in the critical plane; specified wall temperature; etc.) to determine the $M_0^{(1)}(x)$ distribution for the interval up to the right-hand boundary of the characteristic rhomb. The distribution is used to solve the inverse problem for the inviscid flow – that is, to generate the "inviscid nozzle" surface $y^{(1)}(x)$. For the second approximation, on gets,

$$y_w^{(2)}(x) = y(x) + \Delta^{(2)}(x), \quad \Delta^{(2)}(x) = y_w^{(1)}(x) - y^{(1)}(x).$$

This process may be repeated until an n-th distribution $M_0^{(n)}(x)$ is rather close to the function $M_0(x)$ specified.

The present method was employed to design a shaped "viscous nozzle" surface $y_w(x)$ ensuring high-quality flow (M = 18) in the inviscid core at the exit plane of a nozzle with the exit diameter D = 400 mm in a cryogenic vacuum wind tunnel described in [7]. Assumptions include the following: nitrogen as the work gas; total pressure p_0 = 3 MPa; stagnation temperature T_0 = 1500 K; critical cross-section radius y_{w1} = 1 mm; wall temperature T_w = 300 K. In this case the Reynolds number based on exit gas parameters and the nozzle length is less than $0.6 \cdot 10^6$. Flow was assumed to be laminar, and both inviscid and viscous flows were calculated by relations for perfect gases (κ = 1.4). The suitable "viscous nozzle" surface had been generated after two iterations.

Figures 4 and 5 compare theoretical results and experimental data. These are seen to almost coincide.

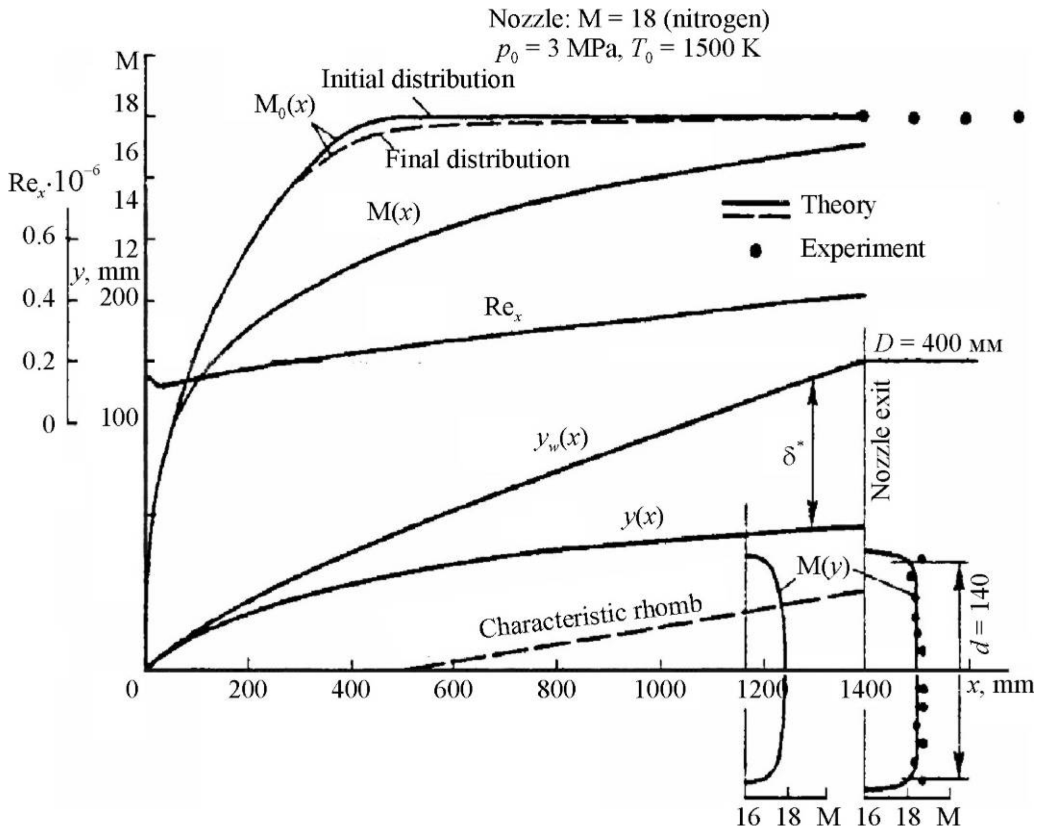

Figure 4. Comparison of theoretical and experimental flows in nozzle at M_p = 18

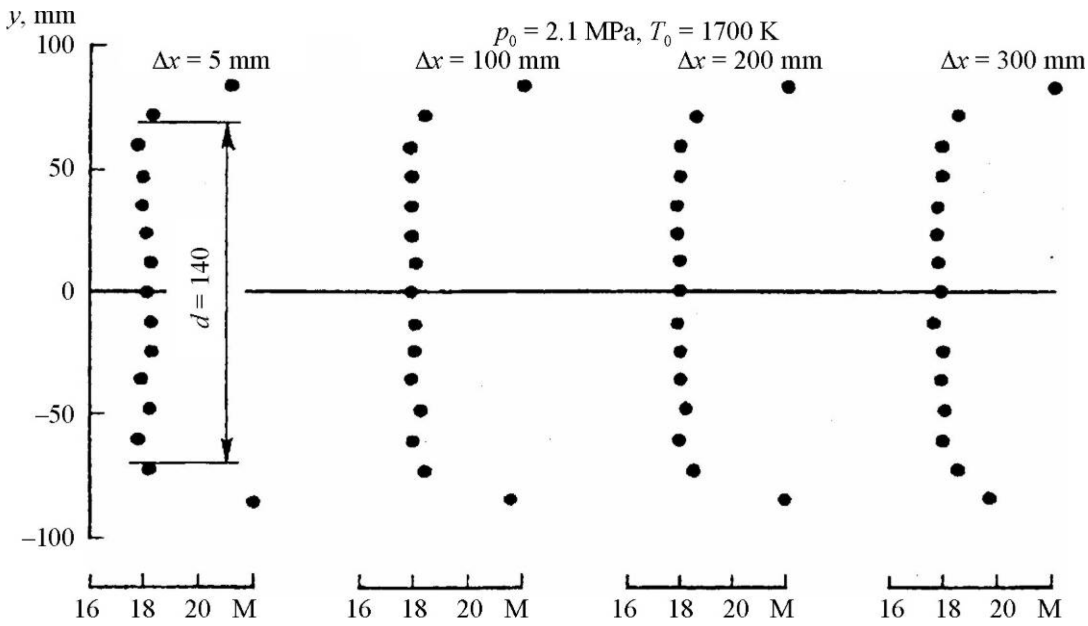

Figure 5. Distributions of M(y) in test section at the specified pressure p_0

Note that $M_0^{(1)}(x)$ after the first approximation is very close to the specification for $M_0(x)$, although δ^* at the nozzle exit plane is as large as the inviscid core radius. This fact is noticeable and can be given the following explanation: in the case of hypersonic flow the gas flowrate in the boundary layer is negligible in comparison with that in the inviscid core, whereas the displacement thickness is equal to the boundary layer thickness.

REFERENCES

1. Mezhirov I.I. Study of flows in hypersonic nozzles of wind tunnels. //Trudy TsAGI, 1981, no. 2119.
2. Byrkin A.P., Verkhovsky V.P., Sosunov A.Yu. Designing the axisymmetric shaped nozzle for M ≤ 10. //Uchenyye Zapiski TsAGI, 1993, vol. 14, no. 2.
3. Verkhovsky V.P., Nosov V.V. Study of flow field in axisymmetric shaped nozzle at M = 7–10. //Uchenyye Zapiski TsAGI, 2004, vol. 35, no. 3–4.
4. Mikhaylov V.V. Method for designing supersonic nozzles with allowance for viscosity. //MZhG, 1969, no. 1.
5. Denisenko O.V. Method for designing supersonic nozzles in the case of notable influence of viscosity. //Uchenyye Zapiski TsAGI, 1992, vol. 13, no. 4.
6. Byrkin A.P., Verkhovsky V.P. On designing the shaped nozzles for hypersonic wind tunnels with allowance for viscosity. //Uchenyye Zapiski TsAGI, 1993, vol. 24, no. 1.
7. Byrkin A.P., Verkhovsky V.P., Zhokhov V.A., Sidorov S.S. Experimental evaluation of flow in shaped hypersonic nozzle of cryogenic vacuum wind tunnel. //Uchenyye Zapiski TsAGI, 2001, vol. 32, no. 3–4.