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MODELLING OF TURBULENT WALL FLOWS UNDER HIGH-FREE-STREAM TURBULENCE INTENSITY AND PRESSURE GRADIENT

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Abstract. For research of the characteristics of the steady and unsteady flows and heat transfer at high-free-stream turbulence intensity the turbulence two-parametric models have been presented. An effect of the free-stream turbulence parameters, their variation along outer boundary layer edge, pressure gradient on the characteristics of flow and heat transfer is analysed. The predicted results with low-Reynolds type models of the laminar-turbulent transition and developed turbulence regime for several test problems are compared with the experimental data.

1. INTRODUCTION

The research based on an analysis of the experimental data[1-6] for the steady near-wall flows at high-free stream turbulence intensity is developed for the unsteady boundary layer driven by a discrete frequency of the external periodic disturbance.

In the modified models for these conditions the fact is used that the processes of the eddy structure interaction of small and large scales in the internal boundary layer region afford the prevailing influence on the wall friction and heat transfer than in the external over-layer. The employment of the proposed modified forms of the wall functions and different approximations for the terms with damping functions of modelling equations is used. The main idea is to develop the most general frame for steady and unsteady flows with the least but sufficient number of parameters incorporated in the turbulent models. The adaptability of the turbulence models at the oscillating time-periodic external flow velocity for the computation of transitional flow and heat transfer characteristics in unsteady boundary layers had been shown in the researches[7,8].

The present paper directs to research of interaction of the large scale perturbations with the small scale wall turbulence on the basis of modified k-ε-models. A combined effect of the free-stream turbulence and harmonic fluctuating velocity parameters on the evolution of transitional and full turbulence under pressure gradients is studied.

2. FORMULATION

The system of the equations for the mean characteristics of time-dependent 2D boundary layer in a compressible flow of homogeneous perfect gas in a system of curvilinear coordinates ξ, ζ, associated with the surface of the body ζ = 0, around which the flow is occurring, has the form

$$\frac{\partial \rho}{\partial t}+\frac{1}{\sqrt{g}}\frac{\partial}{\partial \xi}\left(\rho\sqrt{\frac{g}{g_{11}}}u\right)+\frac{\partial}{\partial \zeta}(\rho v)=0$$

$$\frac{\partial u}{\partial t}+\frac{u}{\sqrt{g_{11}}}\frac{\partial u}{\partial \xi}+v\frac{\partial u}{\partial \zeta}=-\frac{1}{\rho\sqrt{g_{11}}}\frac{\partial p}{\partial \xi}+\frac{1}{\rho}\frac{\partial}{\partial \zeta}\left[\mu\frac{\partial u}{\partial \zeta}-\rho\langle u'v'\rangle\right]$$

$$\frac{\partial h}{\partial t}+\frac{u}{\sqrt{g_{11}}}\frac{\partial h}{\partial \xi}+v\frac{\partial h}{\partial \zeta}=\frac{1}{\rho}\frac{\partial}{\partial \zeta}\left[\frac{\lambda}{c_p}\frac{\partial h}{\partial \zeta}-\rho\langle h'v'\rangle\right]+\frac{1}{\rho}\frac{\partial p}{\partial t}+\frac{u}{\rho\sqrt{g_{11}}}\frac{\partial p}{\partial \xi}+ \tag{1}$$

$$+\frac{\mu}{\rho}\left(\frac{\partial u}{\partial \zeta}\right)^2-\langle u'v'\rangle\left(\frac{\partial u}{\partial \zeta}\right)$$

$$\frac{\partial p}{\partial \zeta}=0\ , \qquad p=\rho RT$$

Here u,v are the longitudinal and normal components of the velocity in the ξ , ζ system (coordinates ξ, ζ are directed along the surface and normal to it), g_{11} is the component of the metric tensor g_{ij} (i,j=1,2); $g = g_{11}g_{22}$, for a 2D flow g_{12} =0; p is the static pressure, ρ is the density, T is the temperature, h is the enthalpy, H is the total enthalpy, μ, λ are the coefficients of viscosity and thermal conductivity, c_p is the heat capacity at constant pressure, R is the gas constant, the subscripts e and w refer to values on an outer edge of the boundary layer and at the wall, t relates to a turbulent regime. The static pressure p is a function of t, ξ. It is assumed that $\mu = \mu_e (h/h_e)^{\omega}$, $\lambda = \lambda_e (h/h_e)^{\omega}$, ω = 0.75.

The boundary conditions are assigned at the surface and outer boundary layer edge:

$$\zeta=0: u=0, \rho v=(\rho v)_w=F(t,\xi),\quad h_w H_0^{-1}=i_w(t,\xi)\ \text{or}\ q_w=q_w^o(t,\xi) \tag{2}$$

$$\zeta\to\infty, u\to u_e, h\to h_e \tag{3}$$

The unsteady boundary layer is considered in which at time $t=0$ the flowfield is given by steady-state conditions; at $t>0$ the external velocity $u_e(t,\xi)$ begins to fluctuate from the steady-state velocity $u_0(\xi)$ on the harmonic distribution

$$u_e(t,\xi)=u_0(\xi)(1+A_0\cos\omega t) \tag{4}$$

The initial conditions on time t are imposed in common case:

$$t=0, u(0,\xi,\zeta)=u_0(\xi,\zeta), h(0,\xi,\zeta)=h_0(\xi,\zeta)$$

Here, u_0, h_0 are the initial given component of the velocity and enthalpy at $t=0$.

The initial conditions for the velocity component and enthalpy profiles on the coordinate ξ are specified in a certain domain, for example at $\xi=\xi_0$

$$u(t,\xi_0,\zeta)=u_{00}(t,\zeta), h(t,\xi_0,\zeta)=h_{00}(t,\zeta)$$

Then at this plane $\xi=\xi_0$ the time-dependent task with the distribution (4) must be solved.

3. MODELLING OF TURBULENCE

At the turbulence flow regime the averaged equation system of a unsteady boundary layer can be closed by using turbulence models based on the concept of the turbulence viscosity and the Kolmogorov-Prandtl hypotheses. These models subdividing to algebraic and differential are generalized by some manner to time-dependent flows. In numerical predictions quasistationary models with "steady" empirical constants have been developed for practical using since its verification was shown. The introduction of the turbulent viscosity and thermal conductivity coefficients, the use of the Boussinesq gradient transport mechanism hypothesis for turbulent stress ($-\rho\langle u'v'\rangle$) , and the form of the Fourier law for turbulent heat flux ($-\rho\langle h'v'\rangle$) allow to represent the total stress τ and the total heat flux q as

$$\tau=\mu\frac{\partial u}{\partial\zeta}-\rho\langle u'v'\rangle=\mu_\Sigma\frac{\partial u}{\partial\zeta},\qquad -q=\frac{\lambda}{c_p}\frac{\partial h}{\partial\zeta}-\rho\langle h'v'\rangle=\frac{\lambda_\Sigma}{c_p}\frac{\partial h}{\partial\zeta} \tag{5}$$

Here $\mu_\Sigma=\mu+\mu_t$, $\lambda_\Sigma=\lambda+\lambda_t$ are the effective transport coefficients. Then introducing in (1) the laminar and turbulent Prandtl numbers $\mathrm{Pr}=\mu c_p/\lambda$ and $\mathrm{Pr}_t=\mu_t c_p/\lambda_t$ permits to express λ/c_p and λ_t/c_p by the ratios μ/Pr and μ_t/Pr_t.

In order to closed the system of the equations two-parameter K-ε -models will be used and specified the free-stream turbulence level (intensity) Tu_∞ ($\mathrm{Tu}_\infty^2=10^4\cdot 2K_\infty/(3V_\infty^2)$) and scale L_∞, where the turbulence kinetic energy $K=0.5\langle u_i'u_i'\rangle$ is divided by V_∞^2.

For the unsteady 2D boundary layer the equations for the turbulence kinetic energy K и the isotropic part of the dissipation rate $\varepsilon=\varepsilon_k-D$ in the coordinate system ξ ,ζ have the form[9]

$$\frac{\partial K}{\partial t}+\frac{u}{\sqrt{g_{11}}}\frac{\partial K}{\partial\xi}+v\frac{\partial K}{\partial\zeta}=\frac{1}{\rho}\frac{\partial}{\partial\zeta}\left[\mu_{\Sigma,k}\frac{\partial K}{\partial\zeta}\right]+\frac{P_k}{\rho}-\varepsilon_k \tag{6}$$

$$\frac{\partial\varepsilon}{\partial t}+\frac{u}{\sqrt{g_{11}}}\frac{\partial\varepsilon}{\partial\xi}+v\frac{\partial\varepsilon}{\partial\zeta}=\frac{1}{\rho}\frac{\partial}{\partial\zeta}\left[\mu_{\Sigma,\varepsilon}\frac{\partial\varepsilon}{\partial\zeta}\right]+\frac{P_\varepsilon}{\rho}-(D_\varepsilon+E) \tag{7}$$

The coefficients of total viscosities $\mu_{\Sigma,k}$, $\mu_{\Sigma,\varepsilon}$ depend on the viscosities μ, μ_t and the Prandtl numbers σ_k, σ_ε for K и ε ; the terms P_k , P_ε in the explicit form describe the generation processes; D_ε is the dissipative term; the terms D, E express an influence of viscosity on the dissipative effects near a wall and low-local-Reynolds-number regions; g_{11}-the metric tensor component.

The expression of the turbulent viscosity coefficient ν_t determined on second Prandtl-Kolmogorov formula with the damping function f_μ of ζ^+ and $c_3^*(\eta_*)$ in the version of Chien model is introduced

$$\nu_t=c_\mu f_\mu\frac{K^2}{\varepsilon},\quad f_\mu=1-\exp(-c_3^*\zeta^+),\quad \zeta^+=\frac{u_*\zeta}{\nu},\quad u_*=\sqrt{\frac{\tau_w}{\rho}} \tag{8}$$

The suggesting modification of the model here assumes that the constant c_3 included in the damping function f_μ in (8) is changed by the function $c_3^*=C_0'/\eta_*^\alpha$, where $C_0'=c_3\eta_*^\alpha(A_0')$ and α=0.25. The function c_3^* connected with the viscous sublayer thickness η_* which given in the form of the relation from local Reynolds number Re_θ and two parameters A_0', B_0' determined by the parameters of free-stream and its turbulence. Here, as in steady case the empirical dependence of A_0' on Tu_∞ obtained for experimental data and transformed for high-intensity turbulence is used.

At the surface and outer boundary layer edge the boundary conditions are realised

$$\zeta = 0 : K = 0, \varepsilon = 0$$

$$\zeta \to \infty, K \to K_e(t,\xi), \varepsilon \to \varepsilon_e(t,\xi)$$

The functions $K_e(t,\xi)$, $\varepsilon_e(t,\xi)$ at outer boundary layer edge are found for the given distribution $u_e(t,\xi)$ outside of the leading critical point neighbourhood from equation

$$\frac{\partial K_e}{\partial t} + \frac{u_e}{\sqrt{g_{11}}}\frac{\partial K_e}{\partial \xi} = -\varepsilon_e\,, \quad \frac{\partial \varepsilon_e}{\partial t} + \frac{u_e}{\sqrt{g_{11}}}\frac{\partial \varepsilon_e}{\partial \xi} = -c_2\frac{\varepsilon_e^2}{K_e} \qquad (9)$$

The initial conditions on t and ξ for the functions K_e and ε_e are set at initial cuts $t=t_0$ and $\xi=\xi_0$. In common case the following initial conditions on t for the functions K и ε are imposed

$$t = 0, K(0,\xi,\zeta) = K_0(\xi,\zeta), \varepsilon(0,\xi,\zeta) = \varepsilon_0(\xi,\zeta)$$

Here, K_0, ε_0 are given the initial distributions of functions K и ε, they can be obtained from solutions of eqs.(6) and (7) at $t= t_0$ when $\partial/\partial t=0$.
The solutions in the downstream region depend on a choice of initial conditions on ξ. At a laminar flow regime the assignment of the initial functions of $K(t,\xi_0,\zeta)$ and $\varepsilon(t,\xi_0,\zeta)$ may be produced by various ways depending on the problem considered.
To determinate of the solutions of eqs. (9) for outer boundary layer distributions of turbulence $K_e(t, \xi)$ and $\varepsilon_e(t, \xi)$ in the first its steady distributions are defined at $t=t_0$ and $K_e(t_0, \xi) = K_{\xi 0}(\xi)$, $\varepsilon_e(t_0,\xi) = \varepsilon_{\xi 0}(\xi)$

$$K_{\xi 0}(\xi) = K_0\left[1 - \frac{p\varepsilon_0}{K_0} I_{e\xi}\right]^{1/p}, \quad \varepsilon_{\xi 0}(\xi) = \varepsilon_0\left[\frac{K_{\xi 0}}{K_0}\right]^{c_2}, \quad I_{e\xi} = \int_{\xi_0}^{\xi} \frac{\sqrt{g_{11}}}{u_{e0}} d\xi \qquad (10)$$

$$p=c_2-1, \; u_{eo}=u_e(t_0, \xi)$$

Here, the functions are used without the subscript “e”: $K_{\xi 0}(\xi_0)=K_0$, $\varepsilon_{\xi 0}(\xi_0)=\varepsilon_0$.The initial conditions on ξ for the functions K_e and ε_e are set at initial cut $\xi=\xi_0$ by the relations $K_e(t, \xi_0) = K_{t0}(t)$, $\varepsilon_e(t, \xi_0) =\varepsilon_{t0}(t)$. After integration characteristic equation and using the initial conditions on ξ the common solutions for the functions K_e and ε_e are obtained

$$K_e = \Phi(t,\xi,K_0,\varepsilon_0,u_e,g_{11}), \; \varepsilon_e = \psi(t,\xi,K_0,\varepsilon_0,u_e,g_{11}) \qquad (11)$$

Applying the partial form of the expression for the velocity on the outer boundary layer edge the solutions (11) may be simplified. The unsteady boundary layer is considered in which at time $t=0$ the flowfield is given by steady-state conditions; at $t>0$ the external velocity $u_e(t,\xi)$ begins to fluctuate from the steady-state velocity $u_0(\xi)$ on the harmonic distribution

$$u_e(t,\xi) = u_0(\xi)(1 + A_0 \cos \omega t) \qquad (12)$$

Here, a number of variants of variable Prandtl number turbulence model and as Pr_t=const are used in computations.

4. RESULTS

In the numerical predictions for Reynolds number $Re_\infty = 0.2835 \cdot 10^6$ ($Re_\infty = V_\infty D/\nu_\infty$, D -

flat plate length) values of Tu_∞ were varied over the range 3-8% and dimensionless $\varepsilon'_\infty(\varepsilon'_\infty=\varepsilon_\infty D/V_\infty^3)$ respectively over $0.184 \cdot 10^{-4}$- 0.184. For the subsonic compressible flow with the moderate velocity value of V_∞ =5.6 m/c and low number M_∞= 0.0164 the next variants of a wall boundary condition for the heat influx equation q_w =const and i_w =const are used. The temperature factor i_w is included over the range 0.7-1.3 ($i_w= h_w/H_0$) and h_e=const. An effect of the free-stream turbulence intensity Tu_∞ on the transition location (the end) determined as the critical Reynolds number $Re_{\theta,*}$ is taken into account in corresponding with the experimental data.

The numerical results obtained with a using of the proposal variant of K-ε model are compared with the experimental data[4] at Tu_∞ =4.86% and different values of ε'_∞.

In the beginning part of flow the pressure gradient is assumed zero. Downstream along the wall at $\xi \geq \xi_1$ an effect of the pressure gradient parameter B on the flow and heat transfer characteristics is defined by the velocity function in (12)

$$u_0(\xi) = V_\infty\left[1 + B(\xi - \xi_1)\right]$$

The distributions of the functions $K'_e(\tau, Re_\xi)$ and $\varepsilon'_e(\tau, Re_\xi)$ on the external boundary layer edge as the solutions (11) of the equations (9) depend on meanings of both parameters of free-stream turbulence Tu_∞ and ε'_∞.

The longitudinal component velocity profiles of $u^+= u/u_*$ depending on the normal coordinate ζ^+ across the boundary layer have been compared for different values of the pressure gradient parameter B in full turbulence region at the turbulence intensity Tu_∞= 4.86 % and $\varepsilon'_\infty=0.184 \cdot 10^{-2}$ with the experimental data *4* (B=0*)* and the theoretical relations *1, 3*. The predicted velocity profiles *2* and *5* correspond on Figure 1 the turbulent flow regime at the longitudinal coordinate $\xi' = \xi/D$ = 1.35, and B=0.2 and 0.4. The function $u^+=\zeta^+$ is the upper curve *1*, the lower curve *3* is expressed by the theoretical relation of the logarithmic wall law $u^+ = 5.81\, lg\zeta^+ + 5.1$.

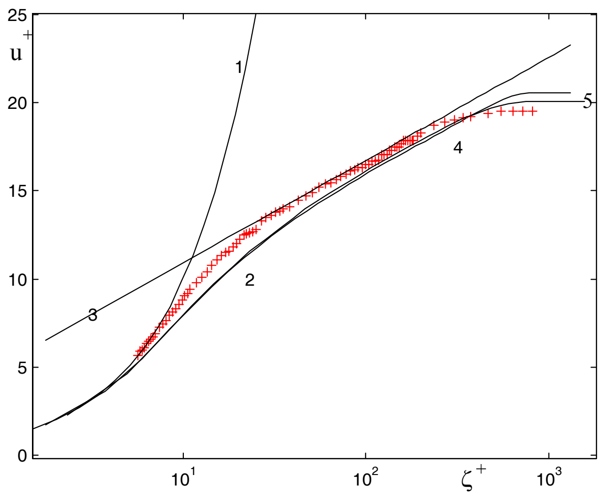

Figure 1. The predicted velocity profiles u^+ depending on the normal coordinate ζ^+ in the full turbulent flow regime at ξ'= 1.35: *1*- $u^+= \zeta^+$; *2, 5* - B=0.2; 0.4; *3* - u^+= 5.81 lgζ^+ + 5.1; *4*- experiment data at $B=0$

The predicted profiles (*2-4*) of turbulence intensity $Q_\tau^+ = K^{1/2}/u_*$ depending on the normal coordinate ζ^+ across the boundary layer are compared on Figure 2 with the experimental data *1* (from the data[4]) for different values of the pressure gradient parameter B at the turbulence intensity Tu_∞= 4.86 %. An increase of parameter B leads to growth of values of the turbulence intensity Q_τ^+ in the wall region of the turbulent boundary layer.

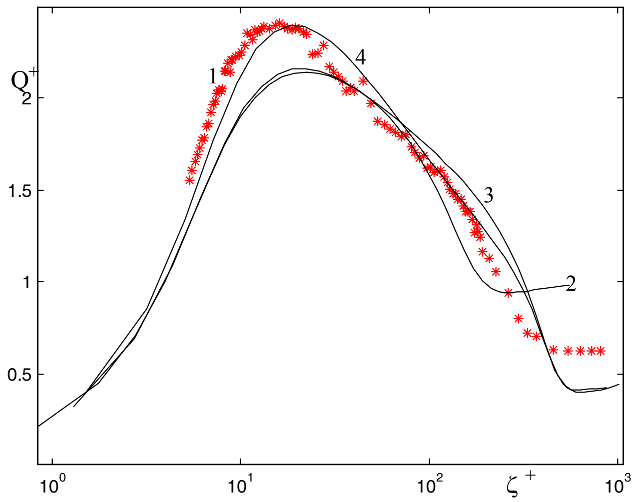

Figure 2: The turbulence intensity distributions of $Q_\tau^+ = K^{1/2}/u_*$ depending on the normal coordinate ζ^+: *1*- experiment data; *2 - B=0,* ξ′=0.6*; 3, 4 - B*=0.2; 0.4, ξ′=1.35

The moderate streamwise pressure gradients intensify the turbulence stresses near wall leading to some increasing of the skin friction coefficient, but downstream these values are decreased and approach to the turbulent empirical line.

The distributions of the heat transfer coefficient St depending on local Reynolds numbers $\mathrm{Re}_{\xi 0}$ for favourable values of the pressure gradient parameter B = 0.2; 0.8 at the turbulence intensity Tu_∞ =4.86 % and dissipation rate $\varepsilon'_\infty = 0.184\cdot 10^{-2}$ are produced in Figure 3. The lines *1, 2* correspond the laminar and turbulent empirical relations. In beginning of the accelerating flow region the values of curves St *(4, 5)* slowly fall below the turbulence line *2*.

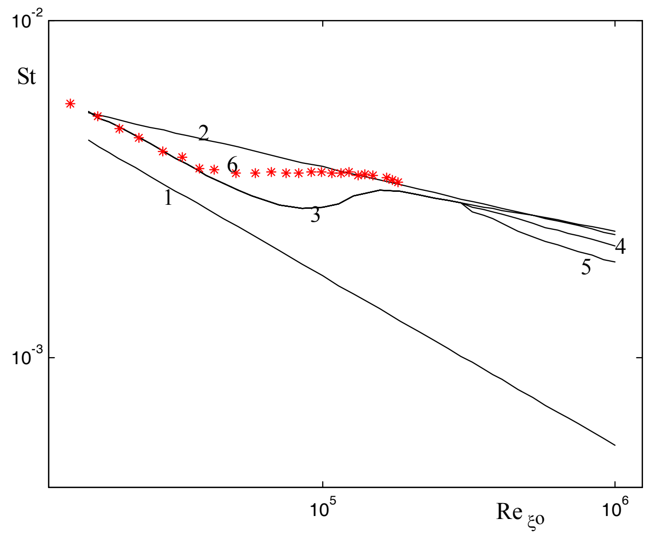

Figure 3: The distributions of number St ($\mathrm{Re}_{\xi 0}$), the lines *1, 2* correspond the laminar and turbulent empirical relations, *3-5- B=0; 0.2; 0.8* are the predicted curves, the experimental data *6*

The space-time distribution of local skin friction coefficient $C_f/2=\tau_w/(\rho_e u_e^2)$ depending on the Re_ξ and τ $(\tau=\omega t)$ for B=0.4 is shown in Figure 4. An increase of the external fluctuation amplitudes A_0 intensifies the time oscillation amplitudes of all the space-time computation characteristics downstream, more significantly in the turbulence region, and leads to quality its evolutions in the transition.

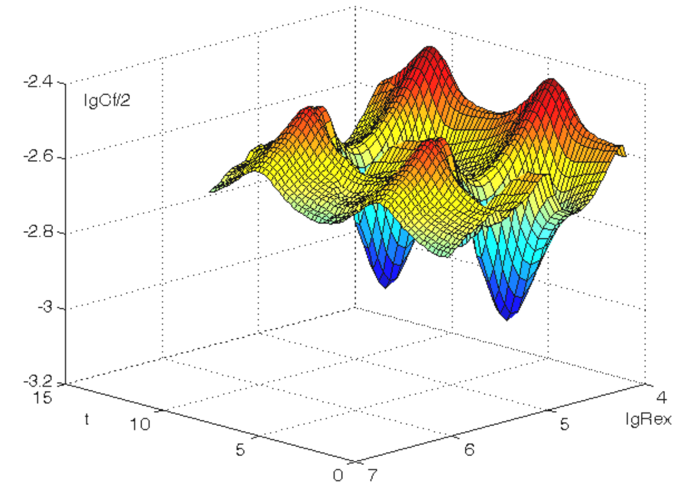

Figure 4. The surface of coefficient $C_f(\tau, Re_{\xi 0})$ at A_0 = 0.352, B=0.4

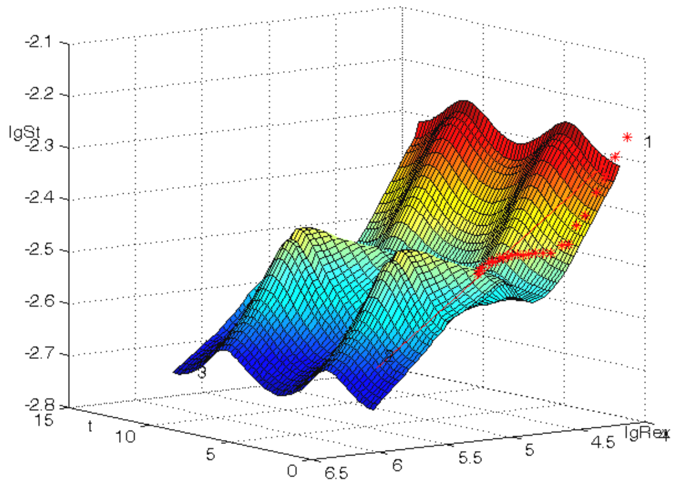

Figure 5. The space-time distribution of Stanton number St(τ, $Re_{\xi 0}$): 1, 2 - the experimental data and turbulence empirical relation at t=0; 3 – the surface of number St depending on t and lg$Re_{\xi 0}$ at the fluctuation amplitude A_0 = 0.352, B=0.4

For the moderate fluctuation amplitudes of the external velocity in the condition of the high free-stream turbulence intensity the distributions of Stanton number St and the temperature factor i_w are transformed from the initial steady-state data in every flow region, especially in the transition, with the conservation of the periodic character and increasing of the oscillating amplitudes for the transition and turbulence flow regimes (Figure 5).

The temperature factor i_w has an increasing tendency in down stream with exception of the transition where its variations on the longitudinal coordinate are not monotone.

5. CONCLUSION

The found properties of the boundary layer at the transition and full developed turbulence under the pressure gradient basically reflect the laws of variations of the averaged values of the flow and heat transfer obtained experimentally. The modification of turbulent models has allowed to make more precise the numerical solutions agreement with the base experimental data. The computation results of the velocity and turbulent intensity profiles, integral boundary layer characteristics correspond to the experiment data.

An analyze of the numerical results has demonstrated the validity of the modified two-parametric model at the research of the oscillating harmonic external velocity effect on the flow characteristics. The regularities of an influence of the free-stream turbulence parameters and fluctuation amplitudes of the external velocity in time have been found under the moderate pressure gradient. With a growth of the flow acceleration parameter an impact of high free-stream turbulence intensity in outer layer on the flow and heat transfer characteristics near a wall is decreased.

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