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COMPUTATION OF COMPLEX UNSTEADY FLOWS AROUND BLUFF-BODIES THROUGH VMS-LES MODELING

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Abstract. The behavior of a variational multiscale LES model is studied for flows past bluff bodies. The circular cylinder is studied for a Reynolds 3900. Subgrid scale eddy viscosities of Smagorinsky, Vreman and the WALE model are compared. The vortex induced motion of a spar equipped with strakes and flexible mooring is then computed. The interest of variational multi-scale as compared to usual Large Eddy Models is put in evidence.

1. INTRODUCTION

Among today's challenges for Large Eddy Simulation, we are interested in the calculation of flows past bluff body involving complex geometry in interaction with turbulence and we consider the application of unstructured meshes. The final purpose is to combine a LES model with a RANS one in a hybrid approach, as in,[46].[1] In the present paper, however, we concentrate on the LES component.

Usual LES subgrid stress modeling like in the Smagorinsky model are based on the assumption of an universal behavior of the subgrid scales. Due to this assumption, energy-containing eddies must not be filtered. Then large Reynolds numbers cannot be addressed with reasonable coarse meshes, except, in particular regions of detached eddies. Even in the case of low Reynolds number or detached eddies, a particular attention must be paid to energetic eddies. For example, the classical eddy-viscosity models are purely dissipative. Often unable to model backscatter, they apply, instead, damping to large resolved energetic eddies. The situation of convolution/differential filters is even worse in the case of irregular meshes, since commutation of the filter with differentiation is generally lost. Looking on the effect of a differential filter, we observe that indeed, we need a good damping of the smaller scale (typically Δx) in order to have a stable and accurate computation, but the directly larger scale (typically $\Delta x/2$) will be also damped. In the case of a second-order differential filter, the strength of the damping will be 4 times smaller for this directly larger scale, and this may be a too large damping, since we wish to reproduce as well as possible the dynamics (transition,...) of the flow under study.

Starting from these remarks, we investigate the application of a complementary mechanism for filtering scales. In the particular case of spectral methods, the principle of Spectral Vanishing Viscosity consists in using the phase space for restricting the action of dissipation, cf.[37–39] In a more general standpoint, the Variational Multiscale (VMS) concept of Hughes[17] appears as a reasonable answer to the filtering issue. The VMS approach was originally introduced by Hughes[17,19] for the LES of incompressible flows and implemented in a Fourier spectral framework using a frequency cutoff for the scale separation (small and large resolved scales). This first VMS-LES version was found to work very well on homogeneous isotropic flows and fully-developed equilibrium and non-equilibrium turbulent channel flows. In this approach, the Navier-Stokes equations are not filtered but are treated by variational projection, and the effect of the unresolved scales is modeled only in the equations representing the small resolved scales. Therefore, no energy is directly extracted from the large structures of the flow. It results in an improved global flow prediction when compared to classical LES models. Later, extensions to the context of finite element methods,[12,14,21,22,27,28,31,32] finite volume methods[9,13,24,41] and the discontinuous Galerkin approach[7] were proposed using different scale separation techniques (filtering, low-order hierarchical finite element basis, and volume agglomeration among others). The VMS-LES method, through these various numerical frameworks, was then applied to complex test cases and industrial flows, for which either the incompressible flow equations or the compressible ones on eventually unstructured grids were considered. Most of the works dealing with VMS-LES have generally considered static eddy viscosity models. Dynamic VMS-LES methods were also investigated in[16,20] with the use of a dynamic viscosity model, and in[8] with a variational interpretation of Germano's identity. Through the numerous studies already achieved in VMS-LES (see also the works[10,11,30,42]), we believe that the VMS concept as introduced by Hughes has already demonstrated its viability and practical utility for the prediction of turbulent flows, and its better competitivity when compared to most traditional LES approaches. The VMS-LES approach (even with simple subgrid scale models as Smagorinsky's model) and dynamic LES models have shown similar order of accuracy, but the former is less computationally expensive and does not require any *ad hoc* treatement (smoothing and clipping of the dynamic constant, as usually required with dynamic LES models) in order to avoid stability problems.

In this work, we consider the VMS-LES implementation presented in[24] for the simulation of compressible turbulent flows on unstructured grids within a mixed finite volume/finite element framwork. We investigate the effect of subgrid scale models in our VMS-LES method for the simulation of a bluff-body flow, and we apply this VMS-LES model to the prediction of a vortex shedding flow around a moving complex geometry.

The remainder of this paper is organized as follows. In Section 2, we summarize the VMS-LES approach used in this study for the simulation of turbulent flows on unstructured grids. Section 3 presents the simulation of a flow around a circular cylinder by the VMS-LES approach with different SGS models. The obtained results are discussed qualitatively and contrasted with those obtained by dynamic LES models. Section 4 reports on the application of the VMS-LES method for the prediction of vortex-induced motion of a complex spar geometry maintained by elastic moorings. Finally, concluding remarks are presented in Section 5.

2. VMS-LES FOR COMPRESSIBLE TURBULENT FLOWS ON UNSTRUCTURED MESHES

In this paper, we consider the *Koobus-Farhat VMS implementation*[24] for the simulation of compressible turbulent flows with a finite volume/finite element discretization on unstructured tetrahedral grids. It uses the flow variable decomposition:[7]

$$W = \underbrace{\overline{W}}_{LRS} + \underbrace{W'}_{SRS} + W^{SGS} \tag{1}$$

where $\overline{W}$ is the large resolved scale (LRS) component of W, W' is its small resolved scale (SRS) component, and W^{SGS} the non-resolved component. The decomposition of the resolved component is obtained by projection onto two complementary spaces $\overline{\mathcal{W}}$ (LRS space) and $\mathcal{W}'$ (SRS space) of the resolved scale space:

$$\overline{W} \in \overline{\mathcal{W}} \quad ; \quad W' \in \mathcal{W}' \ . \tag{2}$$

A projector operator onto the LRS space $\overline{\mathcal{W}}$ is defined by spatial averaging on macro cells, obtained by finite-volume agglomeration which splits the basis/test functions ϕ_l into large scale basis denoted $\overline{\phi}_l$, and small scale basis denoted ϕ'_l.

$$\overline{W} = \sum \overline{W}_l \overline{\phi}_l \quad ; \quad W' = \sum W'_l \phi'_l \tag{3}$$

By variational projection onto $\overline{\mathcal{W}}$ and $\mathcal{W}'$, we obtain the equations governing the large resolved scales and the equations governing the small resolved scales. A key feature of the VMS model is that we set to zero the modeled influence of the unresolved scales on the large resolved ones. This can be interpreted as assuming the locality of interactions in Fourier space. The SGS model is introduced only in the equations governing the small resolved scales, and, by combining the small and large resolved scale equations, the resulting Galerkin variational formulation of the VMS model writes:

$$\left(\frac{\partial(\overline{W} + W')}{\partial t}, \phi_l \right) + \Big(\nabla \cdot F(\overline{W} + W'), \phi_l \Big) = - \Big(\ \tau^{LES}(W'), \phi'_l \Big) \quad l = 1, N \tag{4}$$

where $\tau^{LES}(W')$ is the subgrid scale tensor computed using the SRS component and which is defined by a SGS eddy-viscosity model:

$$\tau_{ij}^{LES}(W') \ = \ -2\mu_{LES}(W') \Big(S'_{ij} - \frac{1}{3} \tilde{S}'_{kk} \Big), \tag{5}$$

with $S'_{ij} = \frac{1}{2} \Big(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \Big)$ the SRS strain rate tensor, and $\mu_{LES}(W')$ the SRS eddy viscosity.

The specific feature of the Koobus-Farhat VMS scale splitting is the definition of the coarse level. It is based on a finite-volume (FV) coarsening, in the sense that FV dual cells are built around every vertex. Then a coarse FV partition is built from agglomeration of a few neighboring fine FV cells. The purpose of this strategy is to restrict LES dissipation inside each coarse cell. In the case of a cartesian mesh, a coarse cell is a cube involving exactly eight cubic fine cells. In the Koobus-Farhat formulation, extension to unstructured meshes is obtained by applying the paradigm of unstructured agglomeration. Once this coarsening is built, a finite-element coarse grid basis is *a posteriori* deduced from it. We refer to[24] for further details.

The VMS technique is combined with a differential LES model. This association carries some control of the dissipation effect of the LES model. However, we shall stress that special attention should still be paid to the choice of the LES model. For the purpose of this study, three SGS eddy-viscosity models are considered: the classical model of Smagorinsky,[44] and two recent and promising models, namely the WALE model[33] and the one of Vreman.[49] The two last SGS models were chosen because -first their eddy-viscosity modeling is based on first order derivatives of the velocity, so that they are rather simple and well-suited for engineering applications, and -second they introduce only a small amount of turbulent dissipation for near-wall flows and transitional flows. This property allows these models to predict more properly near-wall flows, and it allows also turbulence to naturally develop in transitional flow regions.

We now describe these three eddy-viscosity models (hereafter $\tilde{W} = \overline{W} + W'$ denotes the resolved component of W):

In the *Smagorinsky model*,[44] $\nu_{LES}(\tilde{W}) = C_s \Delta^2 \sqrt{\tilde{S}_{ij}\tilde{S}_{ij}}$, where C_s is the constant model set to 0.1, $\tilde{S}_{ij}$ is the strain-rate tensor and Δ measures the local mesh size.

The eddy viscosity in the *Vreman's model*[49] is defined by:

$$\nu_{LES}(\tilde{W}) = c\left(\frac{B_\beta}{\alpha_{ij}\alpha_{ij}}\right)^{\frac{1}{2}} \tag{6}$$

with

$$\alpha_{ij} = \partial \tilde{u_j}/\partial x_i$$

$$\beta_{ij} = \Delta^2 \alpha_{mi}\alpha_{mj}$$

$$B_\beta = \beta_{11}\beta_{22} - \beta_{12}^2 + \beta_{11}\beta_{33} - \beta_{13}^2 + \beta_{22}\beta_{33} - \beta_{23}^2$$

The constant $c \approx 2.5 C_s^2$ where C_s denotes the Smagorinsky constant.

The eddy-viscosity in the *WALE model*[33] is defined by:

$$\nu_{LES}(\tilde{W}) = C_w \Delta^2 \frac{(\overline{S}^d_{ij}\overline{S}^d_{ij})^{\frac{3}{2}}}{(\overline{S}^d_{ij}\overline{S}^d_{ij})^{\frac{5}{2}} + (\overline{S}^d_{ij}\overline{S}^d_{ij})^{\frac{5}{4}}} \tag{7}$$

where

$$\overline{S}^d_{ij} = \frac{1}{2}(\overline{g}^2_{ij} + \overline{g}^2_{ij}) - \frac{1}{3}\delta_{ij}/\overline{g}^2_{kk}$$

is the symmetric part of the tensor $\overline{g}_{ij}^2 = \overline{g}_{ik}\overline{g}_{kj}$, where $\overline{g}_{ij} = \partial\tilde{u}_i/\partial x_j$ and in which the constant C_w is set to 0.1.

In all these eddy-viscosity models, the local mesh size for an element l is defined as follows:

$$\Delta^{(l)} = Vol_l^{1/3} \tag{8}$$

where Vol_l is the volume of the $l-th$ grid element.

Lastly, the numerics used for our compressible flow simulation may have also an important impact on the quality of the prediction. With the compressible Navier-Stokes model, the LES dissipation term does not apply to the continuity equation, so that it is not efficient for mastering acoustics wave stability, and thus it is not sufficient for stabilizing the computation. We use an upwind scheme stabilized by a model of sixth-order dissipation based on the Roe flux splitting. The sixth-order option concentrates the dissipation effect on the smallest scales, reinforcing the main standpoint of these investigations. Details on the numerical ingredients used in the present work can be found in.[5]

3. FLOW PAST A CIRCULAR CYLINDER

In this section, we apply our VMS-LES methodology to the simulation of a flow past a circular cylinder at Mach number $M_\infty = 0.1$ and at a subcritical Reynolds number, based on body diameter and freestream velocity, equal to 3900.

The computational domain, as shown in Figure 1 is: $-10 \leq x/D \leq 25$, $-20 \leq y/D \leq 20$ and $-\pi/2 \leq z/D \leq \pi/2$, where x, y and z denote the streamwise, transverse and spanwise direction respectively. The cylinder of unit diameter is centered on $(x, y) = (0, 0)$.

The flow domain is discretized by an unstructured tetrahedral grid which consists of approximatively 2.9×10^5 nodes. The averaged distance of the nearest point to the cylinder boundary is $0.017D$, which corresponds to $y^+ \approx 3.31$.

For the purpose of these simulations, the Steger-Warming conditions[47] are imposed at the inflow and outflow as well as on the upper and lower surface ($y = \pm H_y$). In the spanwise direction periodic boundary conditions are applied and on the cylinder surface no-slip boundary conditions are set.

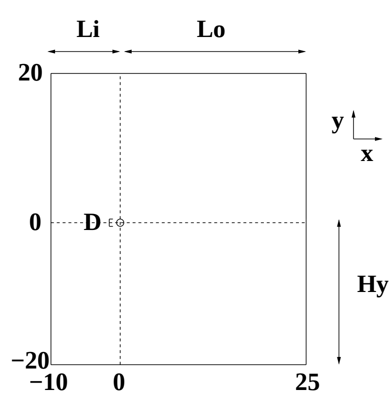


Figure 1: Circular cylinder: Computational domain

To investigate the effect of the different SGS models in the VMS-LES approach, three simulations are carried out on the same grid: VMS-LES with classical Smagorinsky's SGS model as a reference model, VMS-LES with Vreman's SGS model and VMS-LES with WALE SGS model. The preconditioning used for these low-Mach simulations is the Roe-Turkel solver from.[15] The low-diffusion MUSCL scheme stabilized with sixth-order spatial derivatives[5] is used with a numerical viscosity parameter γ set to 0.3.

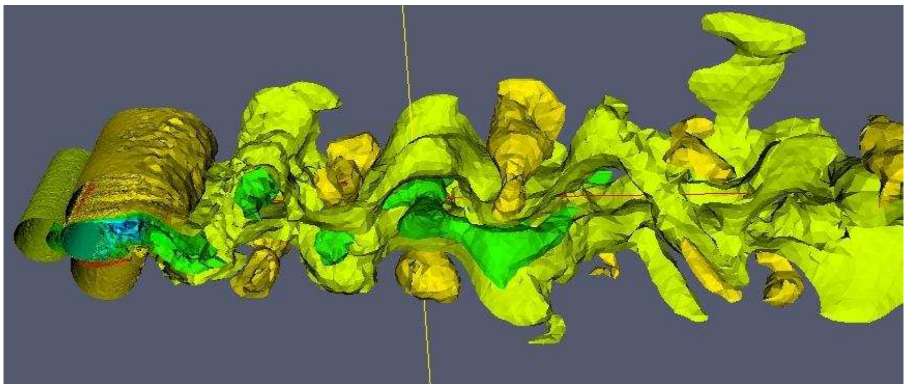


Figure 2: Circular cylinder: Instantaneous streamwise velocity

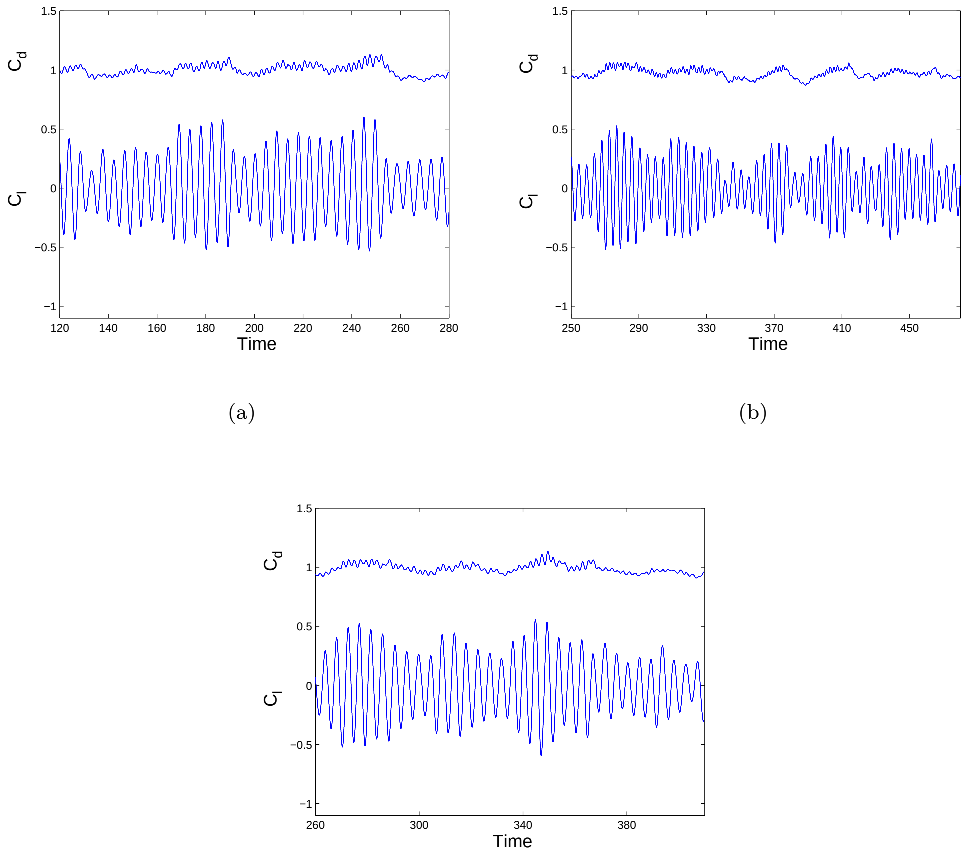
The flow equations are advanced in time with a second-order time accurate implicit scheme. The CFL number was chosen so that a vortex shedding cycle is sampled in around 400 time steps. The instantaneous streamwise velocity isocontours are plotted on Fig. 2. This plot highlights the small structures predicted in the wake by the rather coarse mesh employed in this work, and the three dimensionality of the flow.

The averaged data are obtained using about 27 shedding cycles or 150 nondimensional time unit after the initial transient period. The drag and lift time history obtained in the simulations are shown in Fig. 3.

data from	$\overline{C_d}$	St	l_r	θ_{sep}	$\overline{C_{P_b}}$	U_{min}
VMS-LES Smagorinsky	**1.00**	**0.221**	**1.05**	**88**	**-0.96**	**-0.29**
VMS-LES Vreman	**0.99**	**0.221**	**1.12**	**88**	**-0.91**	**-0.30**
VMS-LES WALE	**0.97**	**0.223**	**1.19**	**89**	**-0.94**	**-0.29**
Numerical data						
25	1.04	0.210	1.35		-0.94	-0.37
4	1.07		1.197	87.7	-1.011	
26	0.99	0.212	1.36		-0.94	-0.33
Experiments						
34	0.99±0.05	0.215±0.05			-0.88±0.05	-0.24±0.1
45				86 ±2		
6		0.215±0.005	1.33±0.05			
36		0.21±0.005	1.4±0.1			-0.24±0.1
29			1.18±0.05			

Table 1: Circular cylinder: Bulk coefficients, comparison with experimental data and with other simulations in the literature. $\overline{C_d}$ denotes the mean drag coefficient, St the Strouhal number, l_r the mean recirculation length: the distance on the centerline direction from the surface of the cylinder tot he point where the time-averaged streamwise velocity is zero, θ_{sep} the separation angle, $\overline{C_{P_b}}$ the mean back-pressure coefficient and U_{min} the minimum centerline streamwise velocity.

Time-averaged values and turbulence parameters are summarized in Table 1 and compared to data from experiments and to numerical results obtained by other



(a) (b)

(c)

Figure 3: Circular cylinder: Lift and drag time history (a) VMS-LES Smagorinsky (b) VMS-LES Vreman (c) VMS-LES WALE

investigators using dynamic LES models on finer grids (containing between half a million and 7.5 million nodes).

As shown in Table 1, the simulations are in good agreement with the experimental values and we notice that the SGS model does not seem to have an impact on the solution accuracy.

Fig. 4 shows the time-averaged streamwise velocity on the centerline direction of the present simulations. There are in accordance with the experiments of Lourenco and Shih (data from Beaudan and Moin[2]) and Ong and Wallace.[36]

Fig. 5 shows the pressure distribution on the cylinder surface averaged in time and on homogeneous z direction. The results from the simulations are very close each other on the whole cylinder and it appears a deviation with the experimental data from Norberg.[34] The discrepancies visible between 60 and 80 degrees may be explained by the rather coarse grid used.

Fig. 6 displays the total resolved Reynolds stress $\overline{u'u'}$ at $x = 1.54$ using the different SGS models. The numerical results correlate well with the experimental data of Lourenco and Shih (data from Beaudan and Moin[2]). Fig. 7 shows the total resolved Reynolds stress $\overline{v'v'}$ at $x = 1.54$ using different SGS models. The peak is overpre-

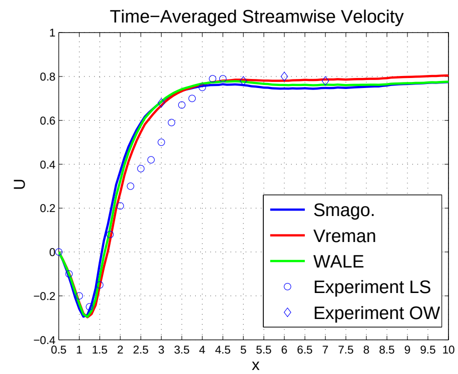


Figure 4: Circular cylinder: Time averaged and z-averaged streamwise velocity on the centerline direction, experiments: Lourenco and Shih[29] (LS) and Ong and Wallace[36] (OW)

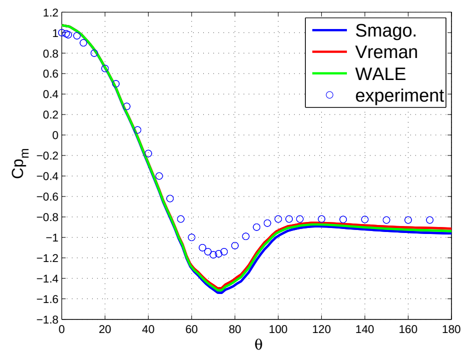


Figure 5: Circular cylinder: Time averaged and z-averaged pressure distribution on the surface of the cylinder, experiments: Norberg[34]

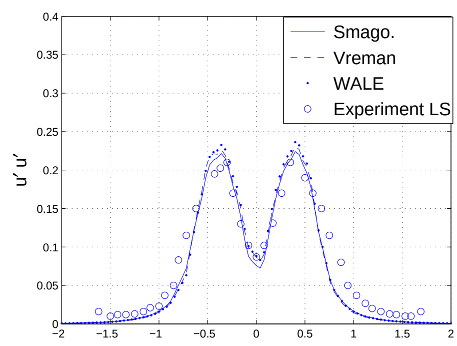


Figure 6: Circular cylinder: Total resolved streamwise Reynolds stress $\overline{u\prime u\prime}$ at $x = 1.54$, experiments: Lourenco and Shih[36] (LS)

dicted by the Smagorinsky's SGS model and underpredicted by the Vreman's SGS model. The WALE SGS model gives the best results. The Fourier energy spectrum of the spanwise velocity at $\mathrm{P}(3, 0.5, 0)$ for VMS-LES with Smagorinsky, Vreman and WALE SGS models is shown in Fig. 8 (a). The frequency is nondimensionalized by the Strouhal shedding frequency. We notice from this figure that the SGS model has no significant influence on the energy spectrum. The Fourier energy spectra

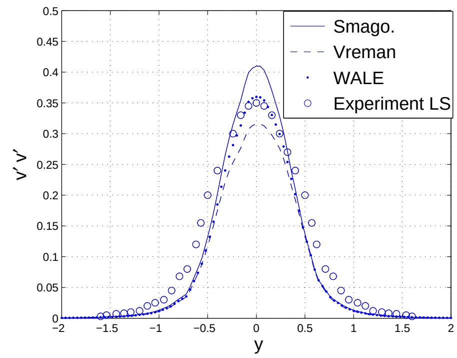
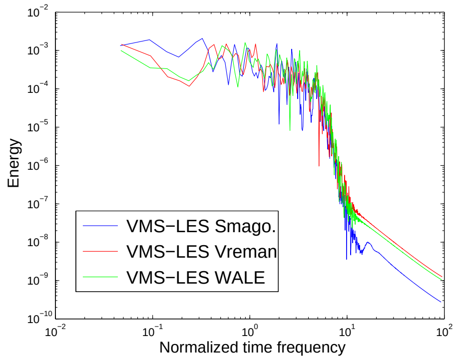


Figure 7: Circular cylinder: Total resolved tranversal Reynolds stress $\overline{v\prime v\prime}$ at $x = 1.54$, experiments: Lourenco and Shih[36] (LS)

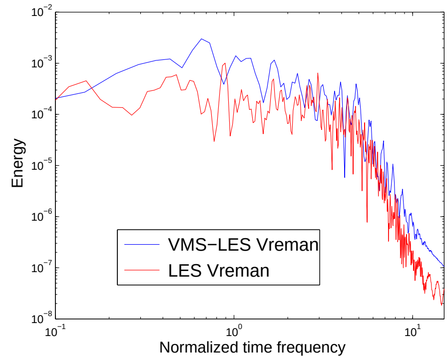
of the spanwise, streamwise and transversal velocity at $P(3, 0.5, 0)$ for Vreman SGS model with LES and VMS-LES are displayed in Fig. 8 (b), (c) and (d) respectively. Via the Taylor hypothesis of frozen turbulence (which is justified since the mean convection velocity is large at that point) which allows to assume that high (low) time frequencies correpond to small (large) scale in space, we observe that the energy in the large resolved scales are higher with VMS-LES than with LES. These results corroborate the fact that in the VMS-LES approach, the modeling of the energy dissipation effects of the unresolved scales affects only the small resolved scales contrary to the LES approach in which these dissipative effects act on all the resolved scales. In Fig. 8 (c) and (d), we also note the peak corresponding to the shedding frequency.

4. VORTEX-INDUCED MOTION OF A COMPLEX GEOMETRY

The prediction of vortex-induced motion of a complex spar geometry is an important motivation for VMS modeling.[43] The spar geometry consists of a cylinder equipped with helicoidal strakes, see Fig.9. Each strake produces in the flow a shear layer that interacts with larger flow structures and inhibits to a large extent the von Kármán vortex street, see Fig.10. This is a typical backscatter effect and we investigate the impact of the choice of a VMS model on the quality of a LES predition. In our computation, the obstacle is maintained by elastic moorings and moves under the effort of the vortex shedding. The fluid-structure coupling is computed at several reduced velocity between 4 and 9 (m/s) and Reynolds numbers between 200000 and 400000. The mesh involves 500000 vertices and the computation is performed during 40 periods before statistics are computed. The behavior of the transverse position of the spar is a key output to be accurately predicted. Fig.11 presents the lateral deviation as a function of time for Smagorinsky and VMS models. The RMS of deviation computed with Smagorinsky is 0.048, and the RMS with VMS model is 0.070, which compares better with the experimental measure, 0.077. Agreement of the VMS calculations with experiments for the different velocitites is demonstrated in Tab.2.



(a)



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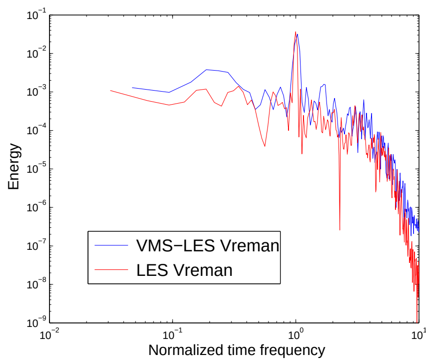
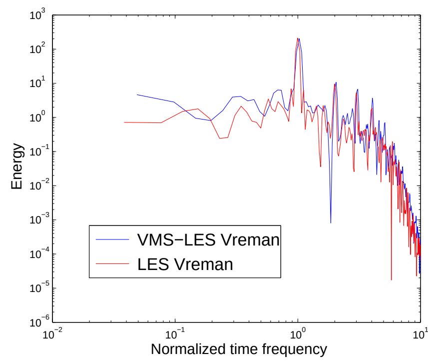


(c)



(d)

Figure 8: Circular cylinder, Fourier energy spectra: (a) spanwise velocity for VMS-LES Smagorinsky, Vreman and WALE, (b) spanwise velocity for LES Vreman and VMS-LES Vreman (c) streamwise velocity for LES Vreman and VMS-LES Vreman (d) transversal velocity for LES Vreman and VMS-LES Vreman

Reduced velocity	4	5	6	7	8	9
LES	-	-	-	.048	-	-
VMS-LES	.0018	.020	.04	.070	.12	.118
Experiments	.0018	.025	.05	.077	.13	.125

Table 2: Vortex-induced motion of a spar: RMS transverse deviation

5. CONCLUDING REMARKS

In this paper, we have presented the application of a Variational multiscale LES model to the prediction of vortex shedding past bluff obstacles. For the case of a smooth body, the model gives correct bulk coefficient and shows that two recently developed SGS models, the Vreman model and the WALE model, combine well inside the VMS formulation. Note, however, that only a slight effect of the SGS model has been observed on the results, and this seems to gives a support to the

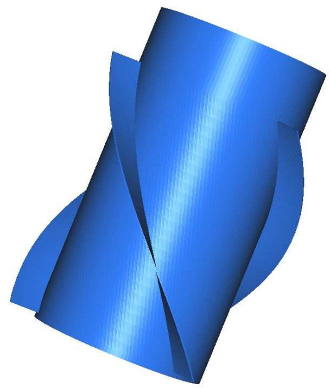


Figure 9: Spar geometry

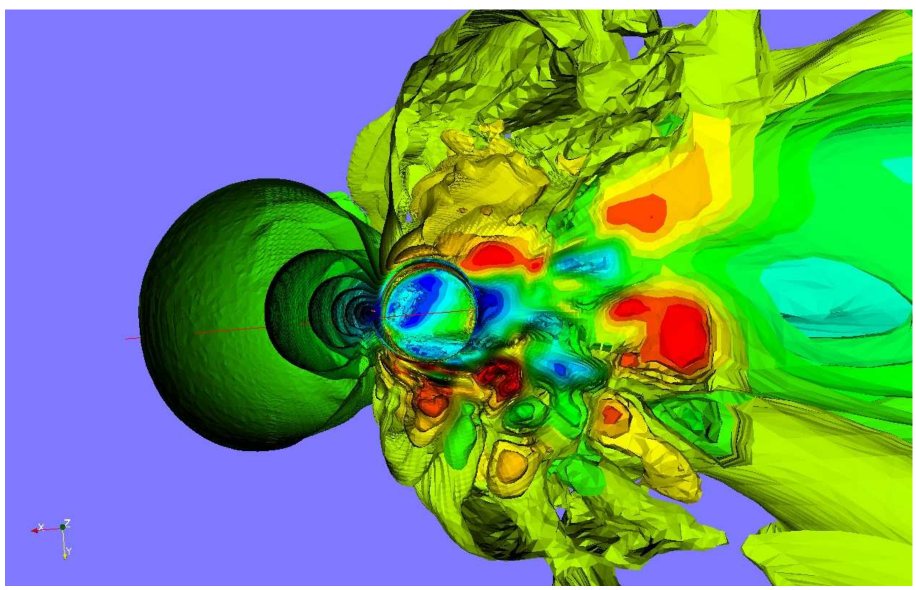


Figure 10: Flow past a spar, Re=300000, VMS-LES with Smagorinsly model, velocity module

VMS idea of adding some dissipation only to the smallest resolved scales. Finally, we remark that our results, obtained on a rather coarser grid and with a second-order numerical scheme, are in good agreement with experimental results, while for "conventional" LES the conclusions of previous studies (see e.g.[25]) was that high order numerical accuracy and hihgly resolved grid are needed to obtain accurate predictions for the considered flow. For the case of a more complex geometry, the VMS capability to transfer energy from small scale to larger ones is demonstrated on the complex fluid-structure interaction describing the vortex induced motion of a flexibly moored spar in a flow. Predictions are again in a good interval. Further investigations will compare the options described in the first part of this study. This work is also connected with the design of a hybrid model combining the VMS formulation with a RANS two-equation one. We refer to the paper of Salvetti and

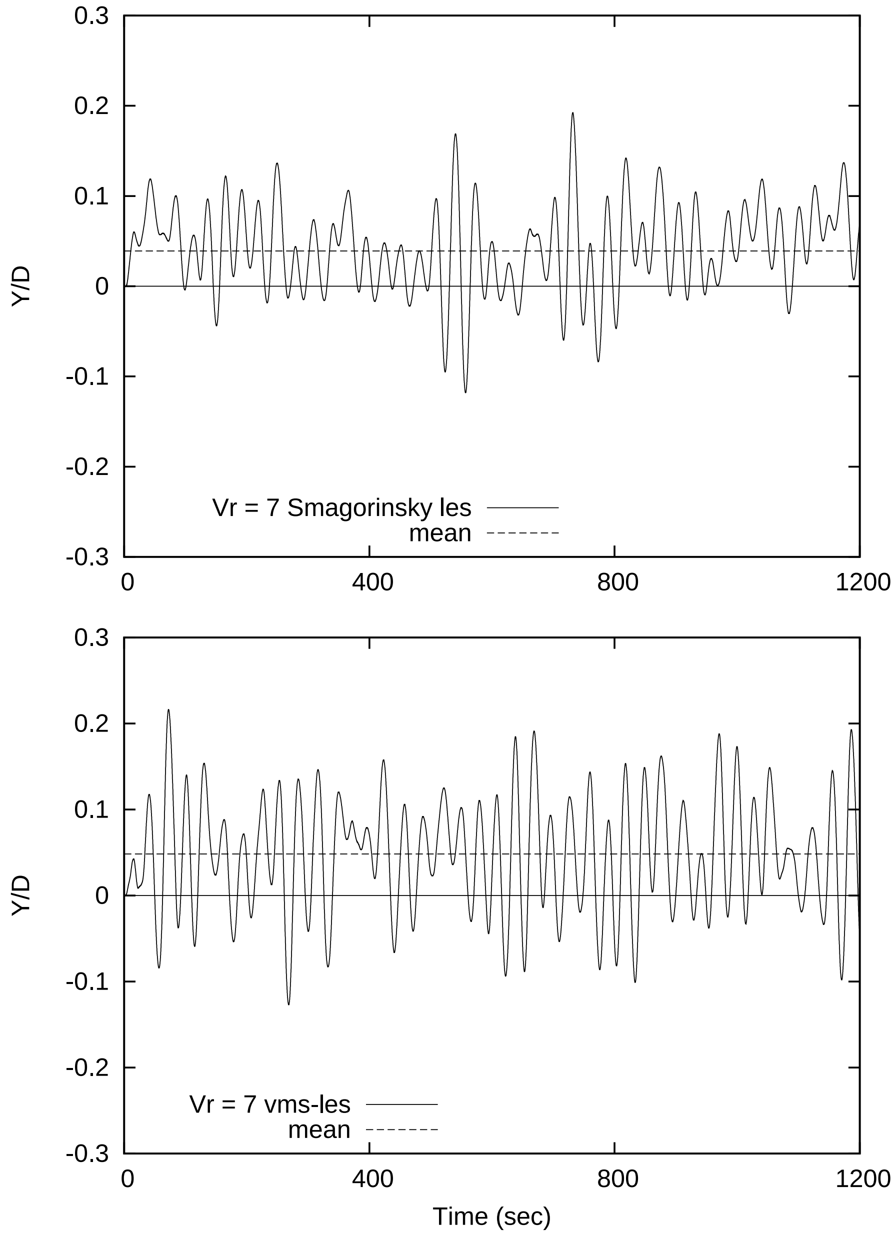


Figure 11: Flow past a spar, spar lateral position as a function of time predicted by pure Smagorinsky (upper curve) and VMS-Smagorinsky (lower curve) models

co-worker in this conference.

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