THREE - DIMENSIONAL HEAT SOURCE IN SUPERSONIC FLOW

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Steady state supersonic gas flow with weak heat source is investigated. The problem of force calculation, which acts on a surface placed near the prescribed heat source, and the problem of heat source boundary location and energy release quantity testing by using pressure distribution measurements through ambient space are under consideration. Peculiarities of three–dimensional flow are described. Characteristic properties of space three-dimensional flow are demonstrated using gaussian spherical source and other forms as an example.

INTRODUCTION

In aerospace problems heat sources play important role inside engines and in tasks for bodies in supersonic flow as concerned to drag reduction, heating, or aircraft control. An electrical discharge, laser radiation, fuel burning, and other chemical reactions can create energy release. [1-8]

Usually it is very difficult to evaluate the completeness of chemical reactions and to define the complex boundaries and energy release intensity distribution by spectroscopic methods. In most cases heat release is defined by flow temperature measurements. But, in this procedure the difficulties also arise. At gas flow temperature measurements a thermometer can measure not only energy transferred to it by neutral molecules of heating gas, but also the energy, which exited radicals release at the thermometer surface. It is necessary to create an alternative method for testing quantity of energy release and geometry (boundary) of heat release area. In the present paper we consider method, based on the gas pressure distribution measurements[5].

1. STATEMENT OF PROBLEM

Let us consider three-dimensional inviscid compressible gas flow with heat source G at speed u_0, pressure p_0 and density ρ_0. System of equation can be written as the following:

$$\frac{\partial \rho u_j}{\partial x_j} = 0, \;\; \rho u_j \frac{\partial u_i}{\partial x_j} + \frac{\partial p}{\partial x_i} = 0, \;\; \rho u_j \frac{\partial}{\partial x_j} \frac{\gamma p}{(\gamma - 1)\rho} - u_j \frac{\partial p}{\partial x_j} = q \qquad (1.1)$$

$$i, j = x, y, z$$

where q is the energy, released at unit volume per unit time, γ is ratio of specific heats. Let us take the value q, which perturbates flow field, small enough, so that perturbed values are

$p = p_0 + p_1$, $\rho = \rho_0 + \rho_1$, $u_j = u_{j0} + u_{j1}$, where $u_{y0} = u_{z0} = 0$, and all values with indicies 1 are also small. Linearization of equations (1.1) gives us

$$\rho_0 \frac{\partial u_{j1}}{\partial x_j} + u_0 \frac{\partial \rho_1}{\partial x} = 0, \tag{1.2}$$

$$\rho_0 u_0 \frac{\partial u_{i1}}{\partial x} + \frac{\partial p_1}{\partial x_i} = 0 \tag{1.3}$$

$$p_0 u_0 \frac{\gamma}{\gamma - 1} \frac{\partial}{\partial x}\left(\frac{p_1}{p_0} - \frac{\rho_1}{\rho_0}\right) - u_0 \frac{\partial p_1}{\partial x} = q \tag{1.4}$$

Equation (1.4) is transformed by using equations (1.2) and (1.3) to the following form:

$$\left(1 - M^2\right)\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{y1}}{\partial y} + \frac{\partial u_{z1}}{\partial z} = \frac{(\gamma - 1)q}{\gamma p_0}, \tag{1.5}$$

where $M = u_0 / a_0$ is Mach number and a_0 is speed of sound in undisturbed flow. Further we denote $\beta = \sqrt{M^2 - 1}$ for brevity.

Let us integrate equations (1.2) - (1.5) by z from $z = -\infty$ to $z = +\infty$ taking into account, that at infinity all perturbations are zero for $x < +\infty$ and $M > 1$. Reference point of coordinates $x = y = z = 0$ is placed inside the heat addition area G (see Figure 1). Then we have:

$$\beta^2 \frac{\partial U}{\partial x} - \frac{\partial V}{\partial y} = -\frac{(\gamma - 1)Q}{\gamma p_0},$$

$$\rho_0 u_0 \frac{\partial U}{\partial x} + \frac{\partial P}{\partial x} = 0, \quad \rho_0 u_0 \frac{\partial V}{\partial x} + \frac{\partial P}{\partial y} = 0, \quad W = 0, \tag{1.6}$$

$$U(x,y) = \int_{-\infty}^{\infty} u_{x1} dz, V = \int_{-\infty}^{\infty} u_{y1} dz, W = \int_{-\infty}^{\infty} u_{z1} dz, P = \int_{-\infty}^{\infty} p_1 dz, Q = \int_{-\infty}^{\infty} q dz,$$

where U, V, P, Q are integrals by z from corresponding values. Derived equations (1.6) are indentical to well known two-dimentional flow equations for all dependent physicall values (integrals by z) U, V, P, Q (see, for example [9]). As a result, the solutions are the same for these two cases for equal $Q(x,y)$ and boundary conditions. But the flow structure for two-dimensional and three-dimensional flows is different. For physical two-dimensional case the value $u_{z1} = 0$ and all other values (u_{x1}, u_{y1}, p_1, q) are constant by z. In general case for physical three-dimensional flow under consideration the equations (1.6) include integrals by z from variable values as functions of z. Below, in contrast to physical two-dimensional flow we assume system of two-dimensional equtaions (1.6) as two-dimensional case.

Reducing P from second and third equations of system (1.6) and using characteristics

$$\xi = x/\beta - y, \qquad \eta = x/\beta + y, \tag{1.7}$$

we find the following relations along the characteristics:

$$V - \beta U = \frac{(\gamma - 1)}{2\gamma p_0} \int_{-\infty}^{\eta} Q d\eta' \text{ along } \xi = const \qquad (1.8)$$

$$V + \beta U = -\frac{(\gamma - 1)}{2\gamma p_0} \int_{-\infty}^{\xi} Q d\xi' \text{ along } \eta = const \qquad (1.9)$$

Equations (1.2)-(1.4) result in:

$$p_1(x, y, z) = -\rho_0 u_0 u_{x1}(x, y, z),$$

$$p_1(x, y, z) = \frac{\rho_0 u_0}{M^2 - 1} \int_{-\infty}^{x} \left[\frac{(\gamma - 1)}{\gamma p_0} q(x, y, z) - \frac{\partial u_{y1}}{\partial y} - \frac{\partial u_{z1}}{\partial z} \right] dx \qquad (1.10)$$

for three-dimensional case and

$$P(x, y) = -\rho_0 u_0 U(x, y),$$

$$P(x, y) = \frac{\rho_0 u_0}{M^2 - 1} \int_{-\infty}^{x} \left[\frac{(\gamma - 1)}{\gamma p_0} Q(x, y) - \frac{\partial V(x, y)}{\partial y} \right] dx \qquad (1.11)$$

for two-dimensional one.

The first equations in (1.10) and (1.11) are valid for all points of flow. The second equations are valid along the characteristics of third groop (lines of stream) $x = var$, $y = const$, $z = const$.

Let us consider only heat addition to flow, that is $q \geq 0$. In this case equations along characteristics (1.8) and (1.9) result in, that $U < 0$, and it follows from the first equation of (1.11), that pressure $P > 0$. These inequalities are valid also for physical two-dimensional case , when real physical pressure perturbation caused by heat release is positive everywhere. In three-dimensional case under consideration this condition may not be valid for real pressure measured, but must be supported for pressure integrals along the lines $x = var, y = const$.

We demonstrate this situation using spherical area of heat release. Let us consider a gaussian heat source distribution:

$$\frac{q(x, y, z)}{q_0} = \frac{exp\left(-\left(x^2 + y^2 + z^2\right) / r_0^2\right)}{\pi\sqrt{\pi}} \qquad (1.12)$$

Here r_0 is a gaussian radius, $q_0 = \Sigma / r_0^3$ and $\Sigma = \iiint_{-\infty}^{\infty} q dx dy dz$ is a summing of heat release in area G. According to equation (1.12) heat addition formally take place up to infinity. We introduce effective radius $R = 2.14 r_0$ of spherical area of the heat release G from the condition $q(R)/q(0) = 0.01$. Numerical solution of system of equation (1.2)-(1.6) was derived using McCormack's scheme (algorithm) of second order of approximation for all coordinates [10].

We placed a reference point at the center of spherical volume G. Let us consider flow picture at the plane $z=0$ (Fig.1) and the plot characteristics $\eta = const$ and $\xi = const$, which are tangential to the energy release area G. It may be seen from equations (1.8) and (1.9), that downstream from characteristics, crossed the point B, values U and V are equal zero and according to the first equation from (1.11), value P is also equal zero. Pressure distribution along x axis (at $y = z = 0$) for a spherical source in three-dmensional p_1 and two-dimensional P cases are shown in Figure 2,a. One can see, that at the space flow pressure p_1 in down stream area from characteristic surfaces, which crossing point B, is not zero, and may be negative. This fact more evidant is demonstrated in Fig.2,*b*, where pressure distribution p_1 is shown along z at the different planes *x*=const for *y*=0. Integrals of p_1 by z at the sections $x = const$, located upstream from point B, are positive, and at the sections $x = const$ down stream from this point are equal zero. Existence of the negative pressure values (more precisely, negative pressure addition owing to heat addition) and existence regions, for which pressure integrals equal zero, are very important peculiarities of three-dimensional flow. Role of these peculiarities will be seen below in the next paragraph. For example, along axis, which crosses the sphere center, downstream the point B, we have

$$\int_{-\infty}^{x} \frac{\gamma - 1}{\gamma p_0} Q(x,0)dx = \int_{-\infty}^{x} \frac{\partial V(x,0)}{\partial y} dx \qquad (1.13)$$

Integral in the left part of equation (1.13) is constant downstream the point A and is positive. Then, and the right integral is positive and positive derivative $\partial V / \partial y$ must dominate in the right integral from point C to B, where equation (1.13) is valid, that is at this interval dominates velocity decrease and dispersion of flow. For spherical area of heat release at Fig.3 distributions (which are included in equations (1.11) and (1.10)) along x coordinate for values $\partial V / \partial y$ and $\partial u_{y1} / \partial y + \partial u_{z1} / \partial z$ are shown. For two-dimensional case as it may be predicted from matter said above just now, the value $\partial V / \partial y$ is positive, and for three-dimensional flow there is a region, in which value $\partial u_{y1} / \partial y + \partial u_{z1} / \partial z$ becomes negative, and this fact demonstrates that region of sign changing arises for perpendicular (radial) component of velocity and there is a region of flow moving to axis. This fact also is related to the existence of negative pressure perturbations.

2. HEAT RELEASE NEAR WALL

In the previous paragraph (subdivision) the flow in open space was considered. Frequently an interaction between heat release area and surrounding surfaces plays an important role. This problem has a double interest. At prescribed intensity of heat release *q(x,y,z)* anyone may be interested in definition the force acted on the surface owing to heat addition. On the other hand, using pressure distributions on the neighboring surface one can define total energy release and shape (boundary) of heat source area.

For example, let us consider an infinite plane $y=y_0$ and area of heat relsease, which includes reference point *x*=*y*=0 (see Fig.1). Moving along the characteristics $\eta=const$ from $\xi = -\infty$ to $\xi = x/\beta - y_0$ on the impervious plane $y=y_0$, we find from equation (1.9):

$$U(x) = -\frac{\gamma - 1}{2\gamma p_0 \beta} \int_{-\infty}^{x/\beta - y_0} Q d\xi \tag{2.1}$$

From this equation one can see that "velocity" $U \le 0$ (deceleration). Then, from the first equation of the system (1.11) it follows, that $P \ge 0$ everywhere on the plate.

Integrating relation (2.1) by x from point D to point E and using the first equation of (1.11), one can derive relation between force F, which influences on plate, and sum Σ of heat released in area G:

$$F = \int_D^E P dx = -\rho_0 u_0 \int_D^E U dx = \frac{(\gamma - 1)\rho_0 u_0}{2\gamma p_0 \beta} \int_D^E dx \int_{-\infty}^{x/\beta - y_0} Q d\xi = \frac{(\gamma - 1) M^2}{\sqrt{M^2 - 1}} \frac{\Sigma}{u_0} \tag{2.2}$$

From equation (1.9) it follows also, that $P(x) = 0$ everywhere on the plate except infinite by z band, located from D to E, that is total force, acted on the plate, is concentrated in infinite band DE. However, it is clear, for example for spherical volume G, that upstream from hyperbola, which arises at plate cross section with Mach cone, having point F as the top (see Fig.1), the flow is undisturbed and $p_1 = 0$. Therefore, real total force is applied only to the part of the band DE, which is cut out by this hyperbola (see Fig.4). For solution of inverse task (calculation of heat released), it is necessarily by using pressure measurements on the plate surface along lines $x = const$, to define those of them, on which $P > 0$. This procedure allows us to define the band DE and using integration of P in the direction of this band, define from equation (2.2) sum of energy released Σ.

These results are typical for two-dimensional task and not taking into account all peculiarities of three-dimensional flow, which as above, are due to negative values of p_1. In Fig.5 pressure distributions $p_1(z)$ on the plate surface are shown for different $x = const$. Along line for $x \ge x_E$ the sum of pressure is $P(x, y_0) = 0$, that is down stream of point x_E positive force acts on the part of area and on the other part acts equivalent negative force. In the band DE below x_E positive force exceeds negative one. Varying the shape of the plate (cutting negative area) we can get the positive force, which is greater than one, influenced on the full plate.

3. OPEN SPACE

For quantity of heat release definition and shape of energy source G research one can replace the plate by static pressure probe. We take a line $(x = x_\alpha, y = y_\alpha, z)$ and measure pressure distribution p_1 along this curve $z = var$ using static pressure probe, which construction and procedure of measuring need for special discussion in every concrete case. Integrating by z this pressure one can find $P(x_\alpha, y_\alpha)$. On the other hand, the value of $P(x_\alpha, y_\alpha)$ one can define moving to the point $(x = x_\alpha, y = y_\alpha)$ along characteristics (1.8) and (1.9). Writing in these equations value U using the value P, we get

$$P(x_\alpha, y_\alpha) = \frac{(\gamma-1)M}{4a_0\beta}\left[\int_{-\infty}^{\xi_\alpha} Q(x(\xi,\eta_\alpha), y(\xi,\eta_\alpha))d\xi + \int_{-\infty}^{\eta_\alpha} Q(x(\xi_\alpha,\eta), y(\xi_\alpha,\eta))d\eta\right] \quad (3.1)$$

$$\xi_\alpha = x_\alpha/\beta - y_\alpha, \quad \eta_\alpha = x_\alpha/\beta + y_\alpha$$

Taking some curve $y = f(x)$ and integrating along it the left and the right parts of the equation (3.1), we derive relation between full power released Σ and integral of measured pressure P along this line.

In particular, taking interval $x_E - x_D$ (at $y = y_0$, see Fig.1), to which at the plate absence perturbations arrive only along characteristics η, as a control curve, we find, taking into account, that second addition (constituent) is zero:

$$\int_D^E P dx = \frac{(\gamma-1)u_0}{4a_0^2\sqrt{M^2-1}}\int_D^E dx \int_{-\infty}^{x/\beta - y_0} Q d\xi = \frac{(\gamma-1)M^2}{2\sqrt{M^2-1}}\frac{\Sigma}{u_0} \quad (3.2)$$

Note, that here value P, measured by static pressure probe, is not equal to P in the formula (2.2), measured by the probe at the plate surface. In the last case it equals, according to (2.1) and (1.11):

$$P(x) = \frac{(\gamma-1)\rho_0 u_0}{2\gamma p_0 \beta}\int_{-\infty}^{x/\beta - y_0} Q d\xi,$$

and for the static pressure probe in an open space value P is one half, according to (1.8)-(1.11) and (3.1).

4. TWO PLATES PARALLEL TO FLOW

There is a situation, when it is impossible to make a static pressure probe at the plane y_0 (for example, it is an aerodynamic tunnel wall, in which impossible to do holes for the probe). In this case the plate may be placed at plane y_1 at the distance $\Delta y = y_1 - y_0$ from the first. It is reliable to place this plate out and upper the heat source, as it can destroy the processes of heat release and pressure measurements. Heat release area may be located at the lower wall.

As at the surface $y = y_0$ flow parameters not measured, than heat release power is related to the pressure, measured at the upper wall, according to the following scheme (Fig.6). Moving from point ***a*** at the upper plate along characteristic η to point ***b*** at the lower plate, where presure is not measured, then moving along characteristic ξ to the point ***c*** at the upper plate. Then we have the following relation:

$$P_c(x) - P_a(x) = \frac{(\gamma-1)\rho_0 u_0}{2\gamma p_0 \beta}\left(\int_a^b Q d\xi + \int_b^c Q d\eta\right) \quad (4.1)$$

Measurement plate can space as far as it possible upstream. But as thriangle ***abc*** moving from $x = -\infty$, then we will have the difference (4.1) equal zero until the one of characteristics of thriangle cross the heat source area. Let us denote by definition corresponding point x_c as x_{on}. Also, the difference (4.1) becomes zero, when both characteristics cancel to cross the heat source area (they leave the heat source area). Let us denote the corresponding point x_c as x_{off}. Integrating equation (4.1) by x and changing independent variables xy in the right side by variables $x\xi$ and $x\eta$, we find:

$$\int_{on}^{off}[P_c(x)-P_a(x)]dx=\frac{(\gamma-1)\rho_0 u_0}{2\gamma p_0\beta}\int_{on}^{off}dx\left(\int_a^b Qd\xi+\int_b^c Qd\eta\right)=2\frac{(\gamma-1)M^2}{\sqrt{M^2-1}}\frac{\Sigma}{u_0}, \quad (4.2)$$

as characteristics ξ and η crossed full heat release area from the beginning to the end when moving along x, that is energy release is twice taking into account. The beginning of heat source «*on*» is called the point, in which one characteristic crossed at first the source, and the end «*off*» is called the point x, at which both charateristics cancel to cross this area. Thus, using by presure measurements at the upper plate, one can define heat release area boundary and its intensity. Similarly one can do measurements using static pressure probe.

In Fig.7,*a* pressure integral (“force”) F variations are shown as a function of longitudinal coordinate x for gaussian source of radius r_0 at M=2, γ=1.4 for three considered cases of open space (curve *1*), single plate (*2*) and two plates (*3*). Analogous results were derived for every shape of source, for example for cylinder of different lengths, stretched along flow (see Fig.7,*b*):

$$\frac{q}{q_0}=\frac{g(x)\exp\left(-\dfrac{y^2+z^2}{r_0^2}\right)}{\pi\left(\sqrt{\pi}+x_1/r_0\right)},\quad g(x)=\begin{cases}\exp\left(-(x/r_0)^2\right), & x<0,\\ 1, \quad 0\le x\le x_1, & \\ \exp\left(-|x-x_1|^2/r_0^2\right), & x>x_1\end{cases} \quad (4.3)$$

CONCLUSIONS

Three – dimensional supersonic flow with heat addition was analyzed. Various variants of force measurements, which impacts on ambient surfaces, were proposed, and also variants of energy quantity calculation and geometry of heat source definition were described using pressure distributions over neighbour surfaces.

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FIGURES

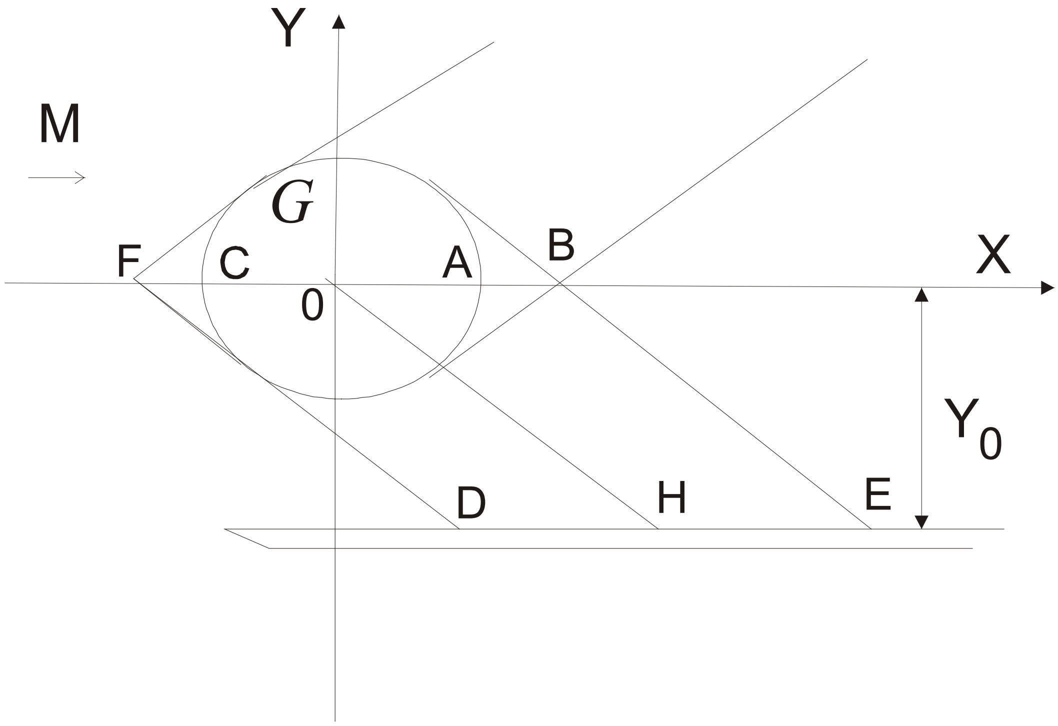


Fig.1. Heat addition area G with spherical shape and reference point at x=0=y=z near the plate, placed at the distance y_0, in supersonic flow. Mach number is M>1. Characteristic radius equals R=2.14r_0 (defined by condition $exp(-(R/r_0)^2)\approx 0.01$), where r_0 is gaussian radius. Cross section at z=0. Characteristic planes (characteristics for two-dimensional) FD, BE are η=*const*, and upper characteristics, which cross points F and B are ξ=*const*.

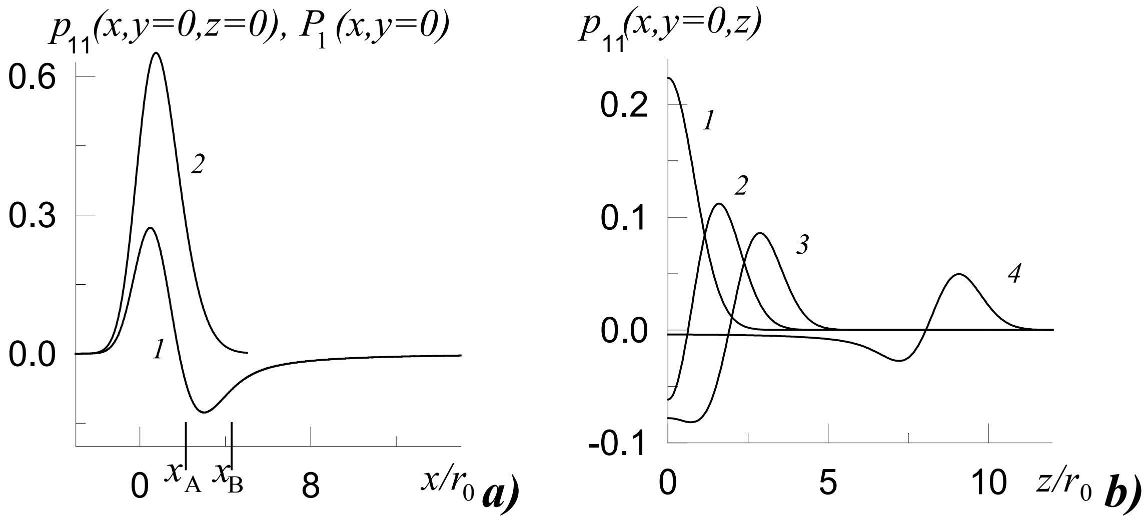


Fig.2,*a)* Pressure distributions along x axis (at $y = z = 0$) for spherical source at three - dimensional $p_{11} = p_1 / \varepsilon p_0$ (curve *1*) and two-dimensional P_1=$P/\varepsilon p_0 r_0$ (*2*) cases; $\varepsilon = (\gamma - 1)\Sigma / \gamma u_0 p_0 r_0^2$ is scale of gasdynamic values perturbations; ***b)*** perturbation pressure distributions $p_{11} = p_1 / \varepsilon p_0$ along z at the difference planes x=*const* and y=0: *1* - x=0, *2* - x_A/r_0=R/r_0=2.14, *3* - x_B/r_0=Mx_A/r_0=4.28, *4* - x=15. Mach number is M=2, γ=1.4 . **Open space** ($y_0 \to \infty$).

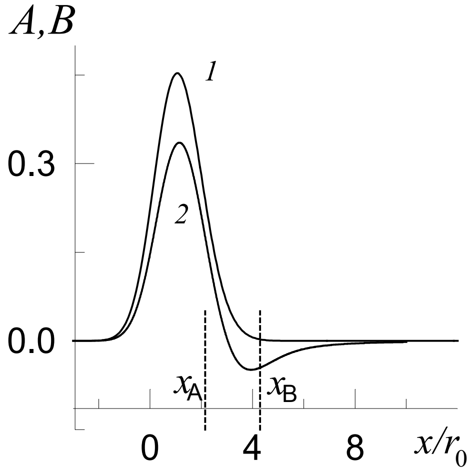
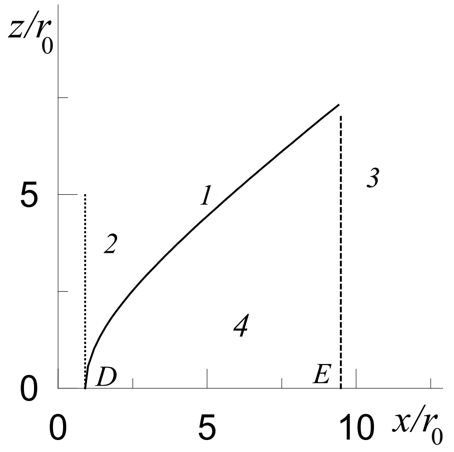


Fig.3. Distributions of values along x for $A = const(\partial V/\partial y)/(\varepsilon r_0 u_0)$ (curve *1*) and $B = const[\partial u_{y11}/\partial y + \partial u_{z11}/\partial z]/(\varepsilon u_0)$ (*2*), where $const = [\gamma M^2/(M^2-1)]$ = 1.67. **Open space.** Mach number is M=2, specific heat capacities ratio is γ=1.4, coordinates $x_A/r_0 = R/r_0$=2.14, $x_B/r_0 = Mx_A/r_0$=4.28 .



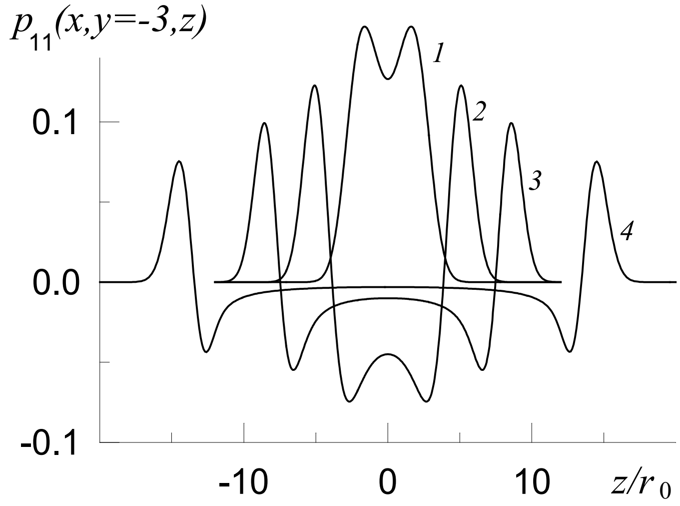


Fig.4. Fig.5.

Fig.4. Cross section of head Mach cone (equation $(x-x_F)^2/(M^2-1)=y^2+z^2$) by the plane $y=y_0$ =const (solid curve, hyperbola *1*). Vertical dashed lines are $x/r_0 = x_D/r_0$=0.908 (curve *2*) and $x/r_0 = x_E/r_0$ =9.5 (*3*). It is recommended to measure perturbative pressure in area *4* **at the plate,** as for $x>x_E$ integral by z of perturbative pressure p_1 is zero.

Fig.5. Perpendicular pressure perturbation distributions $p_{11} = p_1/\varepsilon p_0$ along z **at the plate** y_0/r_0= -3 for different sections:
curve *1* – $x_H/r_0 = |y_0|\sqrt{(M^2-1)}/r_0$=5.2, *2* – $x_E/r_0 = x_B/r_0 + |y_0|\sqrt{(M^2-1)}/r_0$=9.5, *3* – x_E/r_0 = 15, *4* – 25. Sphere, Mach number is M=2, specific heat capacities ratio is γ=1.4.

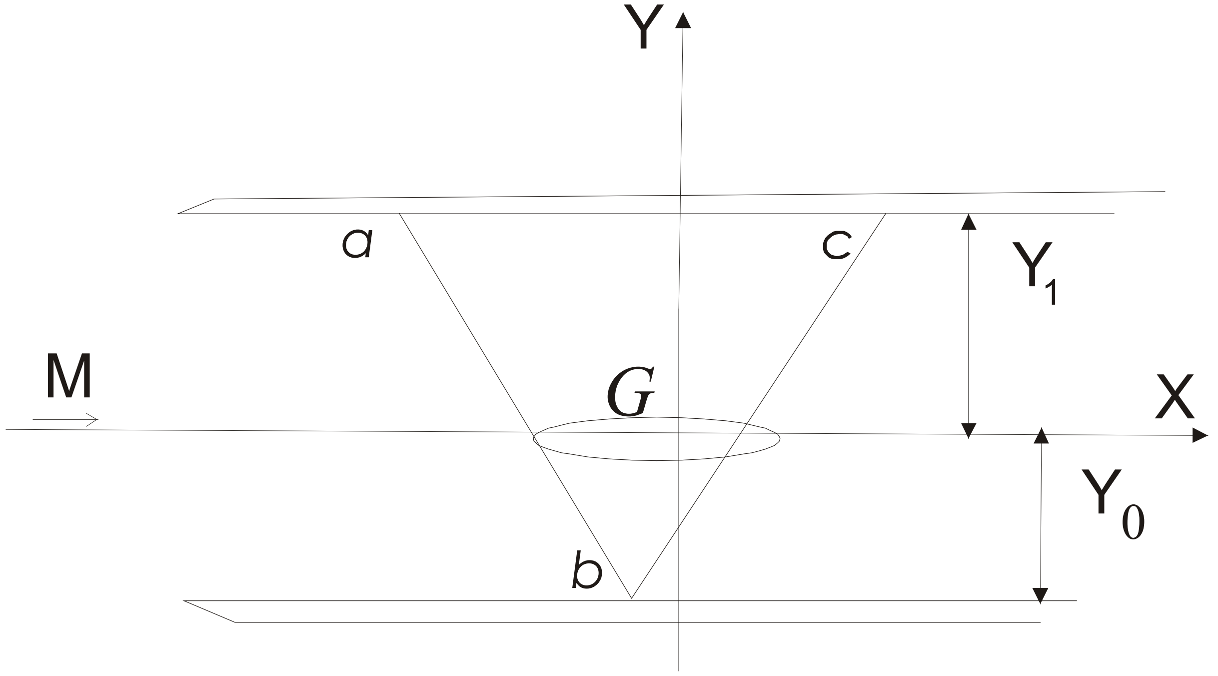


Fig.6. Scheme of pressure measurements at the upper plate for derivation the relation between integral of pressure perturbation F ("force") and the total source power Σ.

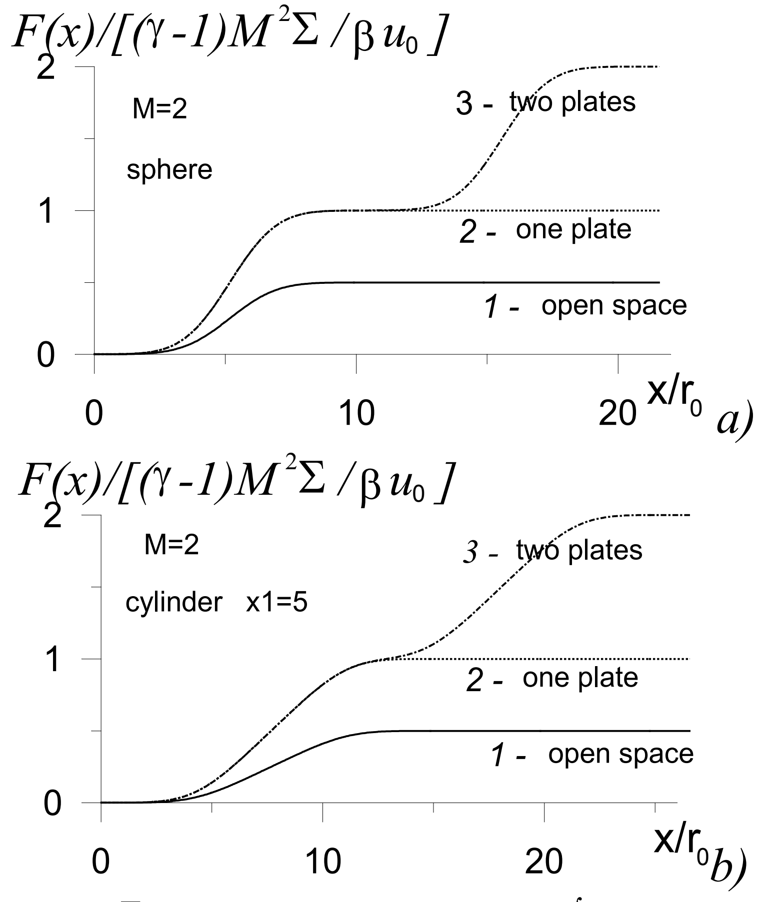


Fig.7. Variations of force F (that is integral of pressure $F(x)=\int Pdx$ along x coordinate) for the cases:

a) spherical gaussian source and *b)* cylinder of length $x_1 = 5r_0$, streched down stream, with gaussian law of intensity diminishing at the boundaries.