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LOW VELOCITYS ANALOGUES OF BASIC COMPONENTS OF HIGH-SPEED FLOWS

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1. Introduction.

One of the most important features of high-speed flows is shock wave, which mark them out from other kinds of flows in the conventional fluid mechanics that are jets, wakes, waves and vortices. As physical properties of media (density, temperature, pressure and velocity) change greatly inside relatively narrow and extended domains, these waves affect drag on moving obstacles and impact negatively on the environment. Being a subject of high-speed flows, shock waves are studied experimentally at expensive and energy consuming experimental facilities. Mathematical description of the flow is constitutive and based on combined schemes operating with the Euler equation for the main flow and Navier-Stokes equation in high gradients domains. The fluid is assumed to be barotropic, and density is constant in an undisturbed space.

But the density of real fluids and gases depends on both pressure and temperature, and concentration of dissolved, or suspended matter, and is distributed non-uniformly in the physical space. Fluids in the environment are stably stratified continuously or discretely. Stratification, even very weak, leads to some new phenomena, which do not exist in a homogeneous fluid. Among them there are inertial and internal waves and so-called 'large scale' and 'fine' structures forming by sets of thin high-gradient interfaces. The interfaces exist for rather a long time with respect the specific diffusion time for their transverse length-scales. It is well known that stratification strongly affects flow separation and downstream wake structure. High gradient interfaces produce picturesque patterns of environmental flows and their laboratory models. For better understanding the flow past obstacles and in order to compare observations with theoretical solutions the stratified flows past obstacles of a simple or perfect shapes, such as strips and right circular cylinders have been investigated. Progress in analysis, computing and in laboratory technique gives enough room for scrutiny theoretical study of the flows basing on exact solutions of governing equations or their linearized residuals satisfying to the real boundary conditions.

The main goal of the paper is to illustrate results by numerical and schlieren visualizations of all components of continuously stratified flow patterns. Quantitative comparison of stratified flows singular components and shock waves in high-speed flows is given, too.

2. Governing equations. Infinitesimal solutions for a flow past a strip.

Dynamics of stratified flows with the density $\rho = \rho\left(S(z)\right)$ is described by a set of fundamental equations including empiric equations of state, which characterizes physical properties of the medium, and differential equations of continuity by D'Alembert, transport of momentum by Navier-Stokes, heat by Fourier and a matter by Fick. In given study the most reduced 3D form of the complete set, which still can be resolved and compared with laboratory experiments, is used. We save only density stratification effects caused by salinity ($S(z)$ is concentration of dissolved or dispersed matter or temperature, appropriate contraction coefficient includes into value S). The undisturbed density profile $\rho(z) = \rho_o \exp\left(-z/\Lambda\right)$ is characterized by scale $\Lambda = \left(d \ln \rho(z)/dz\right)^{-1}$, frequency $N = \sqrt{g/\Lambda}$ and period $T_b = 2\pi/N$ buoyancy, which are supposed to be constant. Then the conventional set of governing equation has the form

$$\rho(S(z))\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}\right)=-\nabla P+\mu\Delta\mathbf{v}+\rho\mathbf{g}, \tag{1}$$

$$\frac{\partial S}{\partial t}+(\mathbf{v}\nabla)S=\kappa_s\Delta S,\ \operatorname{div}\mathbf{v}=0,\ \rho=\rho(S(z))$$

where $\mathbf{v}=(\mathrm{u},\mathrm{v},\mathrm{w})$ is velocity, P is pressure, $\mathbf{g}$ is gravity acceleration, μ and $\nu=\mu/\rho$ are dynamic and kinematic viscosity. This set of non-linear equations with no-slip and no-flux boundary conditions on solid contact surfaces is singular disturbed type with small factors at the terms with the highest derivatives. Complete solutions of these type equations include regular and singular functions depending on small coefficients, that are proportional and inverse proportional to the coefficients. Regular functions describe large components of the flow that are waves and vortices. Singular solutions have not been studied properly.

The problem of the flow around an obstacle in unbounded, incompressible, isothermal, viscous and uniformly stratified fluid is characterised by several dimensional parameters: density $\rho_0(z)$ and its gradient $d\rho_0/dz$, kinematics viscosity ν and salt diffusivity κ_s coefficients, velocity U and size of the body D, and gravity acceleration $\mathbf{g}$. Geometrical parameters of the flow are characterised by intrinsic scales defined by parameters of the set (1) and boundary conditions.

The list of intrinsic length scales of the problem includes the buoyancy scale $\Lambda=|d(\ln\rho_0)/dz|^{-1}$, the size of the obstacle D, the length of the attached (lee) internal waves $\lambda=UT_b=2\pi U/N$, and the length scales of the velocity ($\delta_u=\nu/U$) and density ($\delta_\rho=\kappa_s/U$) boundary layers on the rigid surface. These scales form strong inequalities $\Lambda>>D>>\delta_u>>\delta_s$; $\Lambda>>\lambda>>\delta_u$ for laboratory and environmental conditions. Additional length scales characterise diffusion-induced flows components and singular components of periodic flows in the velocity field $\delta_N=\sqrt{\nu/N}$ and in salinity field $\delta_s=\sqrt{\kappa_s/N}$, and $\delta_\nu>>\delta_s$. Experimental methods must provide visualising the physical fields since a location of the fine interfaces is *a priori* unknown. Plurality of length scales reflects complexity of the flow structure.

In this description of the flow pattern dimensionless parameters, namely the Reynolds number $\mathrm{Re}=UD/\nu=D/\delta_u$, internal Froude number $Fr=U/ND=\lambda/2\pi D$, Peclet number $Pe=UD/\kappa_s=D/\delta_\rho$ and ratio of scales $C=\Lambda/D$ are the ratios of the basic length scales.

For periodic and stationary flows with real positive frequency ω and wave vector $\mathbf{k}=(k_x,k_y,k_z)$ when $\mathbf{v}=\mathbf{v}_0\,\mathrm{f}_p(\mathbf{k},\omega)$, $p=p_0\,\mathrm{f}_p(\mathbf{k},\omega)$, $\rho=\rho_0\,\mathrm{f}_p(\mathbf{k},\omega)$, $\mathrm{f}_p(\mathbf{k},\omega)=\exp(i\mathbf{kr}-i\omega t)$, the general solution of the system (1) can be written as a superposition of plane waves

$$A=\sum_j\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}a_j(k_x,k_y)\exp\left(i\left(k_{zj}(k_x,k_y)z+k_xx+k_yy-\omega t\right)\right)dk_xdk_y \tag{2}$$

where A is a velocity component, pressure, or density. The summation must be performed over *all roots of the dispersion equation* following from the condition of non-trivial solvability of the set (1). Coefficients $a_j(k_x,k_y)$ are defined from the boundary conditions.

If the source is compact plane domain moving with permanent velocity along a rigid plane sloping under the angle φ to horizon, the dispersion equation is one of the 6-th order

$$\left(\omega^2k^2-N^2\left[(k_\xi\cos\varphi-k_\zeta\sin\varphi)^2+k_\eta^2\right]+i\omega\nu k^2\right)(\omega+i\nu k^2)=0 \tag{3}$$

for components of the wave vector in local coordinate frame placed in the centre of the disturbance and $k^2=k_\xi^2+k_\eta^2+k_\zeta^2$. The regular roots of the dispersion equation describe internal waves ahead and past the strip. The singular roots describe edge singularities on the strip as well as internal and isopycnic boundary layers. If strip of length L_x is moving uniformly along the horizontal plane the dispersion relation is simplified and resolved analytically. One of the roots k_i of Eqn. (3) is singular on viscosity and corresponds to boundary layer and the second root k_w is regular on viscosity and corresponds to waves

$$k_{i,w}^2(\omega, k) = -k^2 + \frac{i\omega}{2\nu}\left[1 \pm \sqrt{1 + \frac{4i\nu k^2 N^2}{\omega^3}}\right], \qquad (4)$$

When viscosity is going to zero, regular solutions matches the solutions of Euler equations, and singular solutions are transformed into discontinuities. Solution (3) is substituted in integral for the stream function [1]

$$\Psi(x,z,t) = \frac{iU}{\pi}\int_{-\infty}^{\infty} \frac{1}{k}\sin\frac{k L_x}{2}\; e^{ik(x-Ut)}\,\frac{e^{ik_w(kU,\,k)z} - e^{ik_i(kU,\,k)z}}{k_w(kU,k) - k_i(kU,k)}\,dk \qquad (5)$$

Numerical visualization of solution (5) is presented in Fig. 1. White lines representing loci of points with zero velocity mark edge singularities and external edge of the Prandtl's boundary layer. Shape of phase surfaces of internal waves and points of their contacts with the plane are changed with increasing of the strip length.

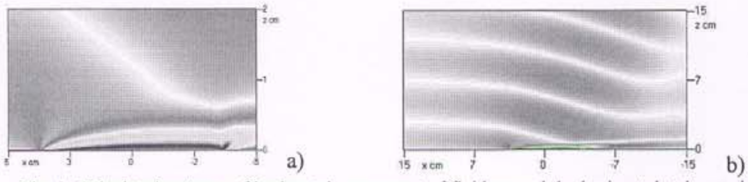

Fig. 1. Calculated patterns of horizontal component of fluid around the horizontal strip moving from right to left along the plane ($T_b = 14$ s, $L_x = 7.5$ cm, $U = 1$ cm/s, $\mathrm{Re} = 7500$, $\mathrm{Fr} = 0.3$, $C = 646$): *a*, *b*) – near and far fields.

3. Experiment

The experiments are conducted in the tank $2.2 \times 0.4 \times 0.6$ m^3 filled from below with a stratified common salt solution using the two-tanks method. The schlieren instrument IAB-458 with different cutting diaphragms registers the side view of the flow pattern. In the experiments the buoyancy period is changed from 7.4 to 17.5 s.

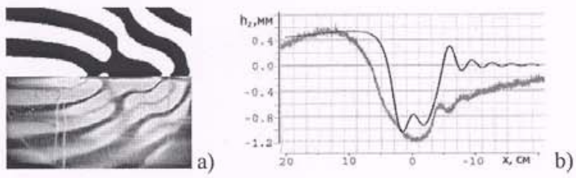

Fig. 2. Flow pattern and particle displacements past strip: $T_b = 7.6$ s, $L_x = 7.5$ cm, $U = 3.97$ cm/s, $\mathrm{Re} = 2780$, $\mathrm{Fr} = 0.63$.

The strip and plastic cylinder tube with length equal to the tank width (40 cm) and the external diameter of 2.5, 5.0 or 7.6 cm as well as strip with length 2.5 or 7 cm is towed horizontally by means of two vertical transparent blades rigidly fastened to a carriage moving along rails mounted above the tank. The towing speed is selected in the range of 0.03 – 6 cm/s, including regimes of wake bubbles and soaring interfaces. Schlieren images of flow induced by strip matched with the calculated flow pattern is shown in left part in Fig. 2, *a*. Calculated and measured by a conductivity probe displacements of fluid particles are shown in Fig. 2, *b*. Differences in shapes of displacement curves reflect impact of non-waves components of motions, which was omitted in the theory. With the velocity increasing singular components are firstly transformed into transverse streaky structure, which are shown in Fig. 3, *a* [2]. Then streaks are gradually condensed into clusters and converted into vortex street (Fig. 3, *b*).

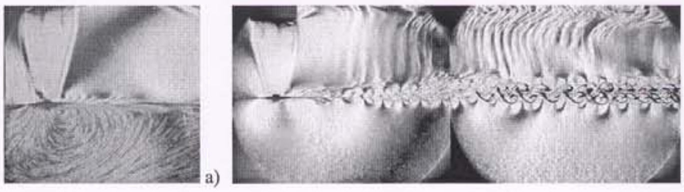
a) b)

Fig. 3. Combined and schlieren images of the flow past strip: $L_x = 2.5$ cm; $T_b = 7.5$ s *a*) – $U = 2.3$ cm/s, *b*) – $U = 5.25$ cm/s

Patterns of observed and calculated far flow field around horizontally moving cylinder are presented in Fig. 4. The body moves from right to left, sloping rays of transient internal waves ahead of the body bound upstream disturbance. Downstream wake with embedded vortices is bounded by thin interfaces. Curved black lines visualize crests and double grey lines are troughs of waves. Positions of embedded vortices as well as crests and troughs of attached internal waves are synchronized. In Fig. 4, *a* and *b* crests and troughs come to the same point on different sides of interfaces in points placed opposite the large vortex bubble inside the downstream wake. The normal components of the flow velocities have different values on different sides if the interface like in shock waves. Internal waves deform the density wake and soaring interfaces. New vortex structures are placed between embedded vortex bubbles. Distinctive vortex pair in Fig. 3, *b* shows that the vortices are produced by the incoming into interfaces fluid.

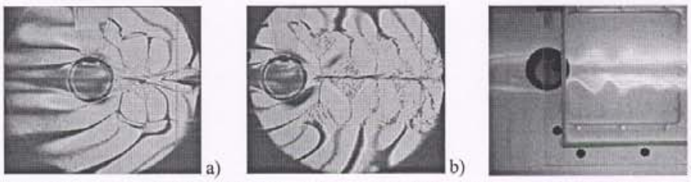
a) b) c)

Fig. 4. Schlieren and dye images of flow induced by horizontal cylinder in a continuously stratified liquid, $T_b = 13$ s, $D = 5$ cm, $U = 0.35$ cm/s; $\mathrm{Re} = UD/\nu = 175$, $\mathrm{Fr} = U/ND = 0.14$: *a*) – forming soaring interfaces; *b*) – vortex systems; *d*) – pattern of dye, $T_b = 7.1$ s, $D = 7.6$ cm, $U = 0.7$ cm/s; $\mathrm{Fr} = 0.1$, $\mathrm{Re} = 530$.

Soaring interfaces that are linear analogue of shock waves accumulate dye from diffuse cloud and transport it with high velocity [3]. Developed complete classification of singular components of periodic flows reveals two distinguished viscous boundary layers [4]. Property of internal boundary layer depends on the slope of the surface. The second boundary layer is Stokes type and does not depend on the problem geometry. All components of flow interact with each other in spite of difference of intrinsic scales. Scrutiny investigation of that flow component provides a new room for study of structure and development of controls methods for high-speed flows.

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References

1. Chashechkin Yu. D. and Bardakov R. N. Two-Dimensional Attached Internal Waves and Concomitant Boundary Layers // Doklady-Earth Sciences. 2004. V. 397. No. 5. P. 677-681
2. Chashechkin Yu.D., Mitkin V.V., Bardakov R.N. Streaky Structures in the Stratified Flow over a Horizontal Strip // Doklady Physics. 2006. V. 51. No. 8. P. 454-458.
3. Chashechkin Yu.D., Mitkin V.V. Transportation of a dye in upstream and downstream wakes of the cylinder in continuously stratified liquid // Journal of Visualization 2007. V. 10. No. 1.
4. Chashechkin Yu.D., Kistovich A.V., Classification of Three-Dimensional Periodic Fluid Flows // Doklady Physics. 2004. V.49. No. 3. P.183-186.

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