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SHOCK WAVE BIFURCATIONS AND SELF-SUSTAINED-WAVES IN VIBRATIONALLY NONEQUILIBRIUM GAS

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Abstract. The influence of vibrational nonequilibrium sustained by an external heat source on the shock wave structure is investigated. The gas with stationary nonequilibrium has the five fields of the nonequilibrium degree S with qualitatively different acoustical properties. Nonequilibrium leads to the strong modification of shock waves. The shock wave adiabat for a nonequilibrium gas with an exponential relaxation law is constructed. Unlike the frozen adiabat, this adiabat has two branches. Possible shock wave structures are described in S –D bifurcation diagram (D is the stationary wave speed). Weak shock waves are unstable. They disintegrate into sequence of the self-sustaining waves. Two types of the self-sustained waves (the pulse and the wave with non-zero asymptote) are obtained.

1. GENERAL SPECIFICATIONS INTRODUCTION

Klimov et al.[1-3]; Basargin and Mishin [4-6]; Bystrov, Ivanov, and Shugaev [7]; Gridin, Klimov, and Molevich [8]; Ganguly, Bletzinger, and Garscadden [9] have investigated the weak shock wave propagation in weakly ionized nonequilibrium plasma. The different types of electric discharges (direct current discharges, radio frequency discharges and pulsed discharges) are used in these experiments. It was observed both the shock wave amplification (in molecular gases, such as air, nitrogen, CO_2) and the shock wave decay. It was also observed the acceleration of shock waves, the precursor generation before the shock wave, the shock wave structure modification, and the shock front splitting. The modification of SW structure was also observed in chemically active gas. For the first time, weak SW amplification (M~1.05) in chemical active mixture $Cl_2 : H_2 : Ar = 1:3:7$ ($P = 0.33\, atm$) was studied by Abouseif, Toong, and Converti [10]. The SW had a sharp front. Secondary waves had smooth fronts. Secondary waves were amplified considerably in chemically active medium.

Bailey and Hilbun [11]; Macheret et al. [12] have tried to explain the shock wave decay, the acceleration of shock waves, and the shock front splitting obtained in the inert gases by the thermal mechanism. Thermal mechanism takes into account non-uniform gas heating in a plasma region and the shock wave front curvature. However, Klimov et al. [13], Mishin, Klimov, and Gridin [14]; Gridin and Klimov [15] have observed the shock wave splitting and plasma precursor generation in pulsed transverse discharges in air, where the high temperature homogeneity and the absence of the shock wave curvature were controlled. Thus, the thermal mechanism can't be responsible for the new shock wave structure in air in these conditions. It can't explain the shock wave amplification at all.

The amplification and the modification of weak shock waves can be caused by the new viscosity-dispersion properties of nonequilibrium media. In [16-18], we have discussed in detail the principle difference between acoustics of equilibrium media and acoustics of such nonequilibrium media as a vibrationally excited gas, a nonisothermal plasma, chemically active mixtures, media with nonequilibrium phases etc. In such media, the second (bulk) viscosity coefficient ξ and the sound dispersion can be negative: $\xi < 0$ and $u_0 > u_\infty$. Here, u_0, u_∞ are the equilibrium (low-frequency) and frozen (high frequency) sound velocities, respectively. Media possessing the negative viscosity are acoustically active. Moreover, the low-frequency coefficient of the gas dynamic nonlinearity Ψ_0 is a complicated function on the nonequilibrium degree. Only the frozen coefficient of gas dynamic nonlinearity has the usual form $\Psi_\infty = (\gamma_\infty + 1)/2$. There are ranges of nonequilibrium degree, where $\Psi_0 < 0$.

In the present work, we investigate theoretically the possible modification of the shock wave structure in stationary nonequilibrium media, which is caused by their new acoustical properties.

2. SHOCK ADIABATS IN NONEQUILIBRIUM MEDIUM

The initial system of gas dynamics equations has the form

$$P = \frac{\rho T}{M}, \quad \frac{d\rho}{dt} + \rho\frac{\partial \mathrm{v}}{\partial x} = 0, \quad \rho\frac{d\mathrm{v}}{dt} = -\frac{\partial P}{\partial x}, \; C_{V\infty}\frac{dT}{dt} + \frac{dE_v}{dt} - \frac{T}{\rho}\cdot\frac{d\rho}{dt} = Q - I,$$

$$\frac{dE_v}{dt} = \frac{E_e - E_v}{\tau_v(T,\rho)} + Q \qquad (1)$$

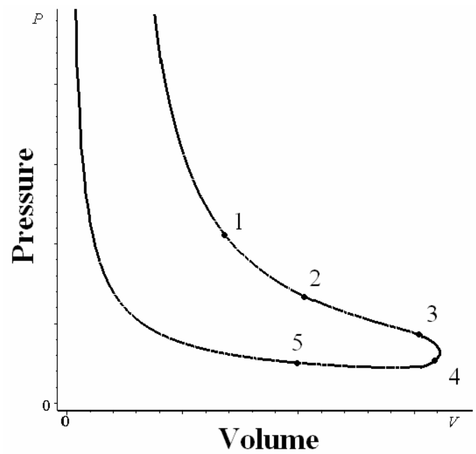

Figure 1. An equilibrium adiabat. Initial states 1-5 corresponds to five different fields of nonequilibrium

In Eq. (1), E_v is the energy of the vibrational degrees of freedom of the molecules, E_e is its equilibrium value, τ_v is the vibrational relaxation time, and Q is the power of an external heat source (in particular, electric pumping in the discharge or optical pumping), sustaining nonequilibrium degree $S=(E_{v0}-E_{e0})/T_0$, v,T,ρ,P are, respectively, the velocity, temperature, density, and pressure, I=Q is the heat removal and $d/dt=\partial/\partial t+v\partial/\partial x$.

The gas with stationary nonequilibrium has the five fields of the nonequilibrium degree S with qualitatively different properties [17-19]:

Field 1: $S<S_{th}=C_v/(C_{V\infty}-\tau_T)$, where $C_v=dE_{e0}/dT_0$, $\tau_T=\partial\ln\tau_{v0}/\partial\ln T_0$. Here, we have the positive second (bulk) viscosity $\xi_0>0$, the positive dispersion $c_0<c_\infty$, and the positive nonlinearity coefficient $\Psi_0\approx(\gamma_0+1)/2$ similar to equilibrium media.

Field 2: $S_{th}<S<S_n$. The dispersion and the second viscosity are negative ($\xi_0<0$; $c_0>c_\infty$). In fields 2-5 media are acoustically active. The low frequency nonlinear coefficient $\Psi_0>0$. Here S_n is defined from the equation $\Psi_0(S_n)=0$, where

$$\Psi_0=\left[\frac{S_0\tau_T(1+S_0)}{C_{P0}C_{V0}}+\frac{1+2C_{V0}}{2C_{V0}}-\frac{S_0(1+S_0)^2}{2C_{P0}C_{V0}^2}\tau_{TT}\right],\tau_{TT}=\frac{T_0^2}{\tau_{v0}}\frac{\partial^2\tau_{v0}}{\partial T_0^{\;2}},$$

$C_{V0}=C_{V\infty}+C_v+S_0\tau_T$, $C_{P0}=C_{P\infty}+C_v+S_0(\tau_T+1)$ are the low-frequency heat capacities at constant volume and constant pressure in the vibrationally excited gas.

Field 3: $S_n<S<S_V=\dfrac{C_{V\infty}+C_v}{-\tau_T}$. Here, $\xi<0$, $c_0>c_\infty$, $\tilde{\Psi}_0=\gamma_0\Psi_0<0$.

Field 4: $S_V<S<S_P$; $S_P=\dfrac{C_{P\infty}+C_v}{-(\tau_T+1)}$. ; Here, $\xi<0$, $\tilde{\Psi}_0>0$, $C_{V0}<0$, $C_{P0}>0$.

Field 5: $S>S_P$. Here, $\xi<0$, $c_0<c_\infty$, $\tilde{\Psi}_0>0$, $C_{V0}<0$, $C_{P0}<0$.

In relaxation gas dynamics, two shock adiabats drawn through a given initial point (P_0,V_0) are considered. One corresponds to total equilibrium of the final states of the gas and, therefore, is called the equilibrium adiabat. The other, referred to as “frozen,”assumes that the relaxation processes do not proceed at all. These adiabats can be obtained from the general Rankine-Hugoniot expression

$$\varepsilon_0-\varepsilon_1+\frac{1}{2}(V_0-V_1)(P_0+P_1)=0,$$

where subscripts 0 and 1 correspond to stationary states before and after the shock front, $V=1/\rho$ is the specific volume, ε is the specific inner energy.

The frozen adiabat corresponds to $\varepsilon_0=C_{V\infty}T_0+E_{v0}$, $\varepsilon_1=C_{V\infty}T_1+E_{v0}$, where $T=MPV$, from which it follows [20]

$$\frac{P_1}{P_0}=\frac{(\gamma_\infty+1)V_0-(\gamma_\infty-1)V_1}{(\gamma_\infty+1)V_1-(\gamma_\infty-1)V_0}.$$

The equilibrium adiabat corresponds to $\varepsilon_0=C_{V\infty}T_0+E_{v0}=(C_{V\infty}+S_0)T_0+E_e(T_0)$, $\varepsilon_1=C_{V\infty}T_1+E_{v1}=(C_{V\infty}+S_1)T_1+E_e(T_1)$.

For Landau-Teller dependence

$$\tau_v(T,\rho) = B\frac{\exp(b/\sqrt[3]{T})}{\rho\sqrt{T}},$$

and an equilibrium vibrational energy in harmonic - oscillator form

$$E_e = \frac{\theta}{\exp(\frac{\theta}{T})-1},$$

where B, b, and θ are constants, we obtain [19]

$$C_{V\infty}(P_0V_0 - P_1V_1) + \frac{1}{2}(V_0 - V_1)(P_0 + P_1) +$$
$$S_0P_0V_0\left[1-\sqrt{\frac{V_1P_0}{V_0P_1}}\exp\left(\frac{b}{\sqrt[3]{MP_1V_1}} - \frac{b}{\sqrt[3]{MP_0V_0}}\right)\right] +$$
$$\frac{\theta}{M}\left\{\left[\exp\left(\frac{\theta}{MP_0V_0}\right)-1\right]^{-1} - \left[\exp\left(\frac{\theta}{MP_1V_1}\right)-1\right]^{-1}\right\} = 0.$$

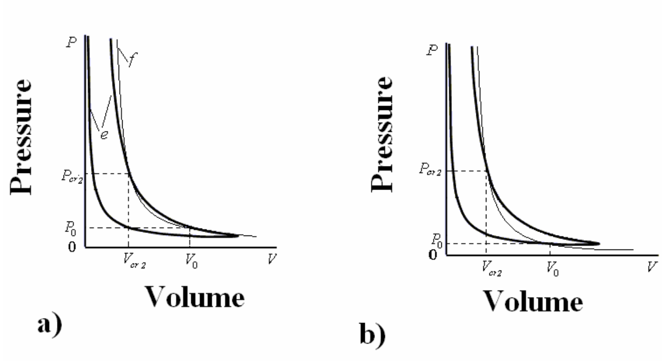

Figure 2. Frozen (f) and equilibrium (e) shock adiabats in nonequilibrium medium [19]. Nonequilibrium degree S: a) field 2, b) field 5.The equilibrium adiabat has two branches and meets with the frozen adiabat in the point (P_{cr2}, V_{cr2}).The point (P_0, V_0) corresponds to an initial state before the shock wave front. The initial point (P_0, V_0) on the equilibrium adiabat moves from upper branch (a) to lower branch (b) with increase in the nonequilibrium degree

For $S_0 \neq 0$, the equilibrium adiabat has two branches with two asymptotes $P \to \infty$ (Figure 1). There is the point (P_{cr2}, V_{cr2}) where the frozen and equilibrium adiabats meet. With increase in the nonequilibrium degree, the initial point (P_0, V_0) on the equilibrium adiabat moves from the upper branch (Figure 2a) to the lower branch (Figure 2b).

3. SHOCK WAVE STRUCTURES. BIFURCATION DIAGRAM

In[19], we reduced system of equations (1) for stationary waves propagating with the speed D to one equation

$$\frac{d\rho}{dz} = -\frac{\rho\{[E_e(\rho) - E_v(\rho)]/\tau_v(\rho) + Q\}}{\rho_0 D(dE_v / d\rho)} \equiv \frac{A(\rho)}{B(\rho)}\ , \qquad (2)$$

$$E_v(\rho) = E_{v0} + M[C_{P\infty}\frac{P_0}{\rho_0} + \frac{D^2}{2} - \frac{C_{P\infty}}{\rho}(P_0 + \rho_0 D^2(1 - \frac{\rho_0}{\rho})) - \frac{1}{2}(\frac{\rho_0 D}{\rho})^2],$$

where $z = x - Dt$.

The shock wave structure after the sharp front ρ_d, which is equal to

$$\frac{\rho_d}{\rho_0} = \frac{(\gamma_\infty + 1)D^2}{(\gamma_\infty - 1)D^2 + 2c_\infty^2}\ ,$$

was obtained using the numerical solution of an equation (2). Integral curves and possible stationary wave solutions of Eq. (2) are shown in Figures 3-7.

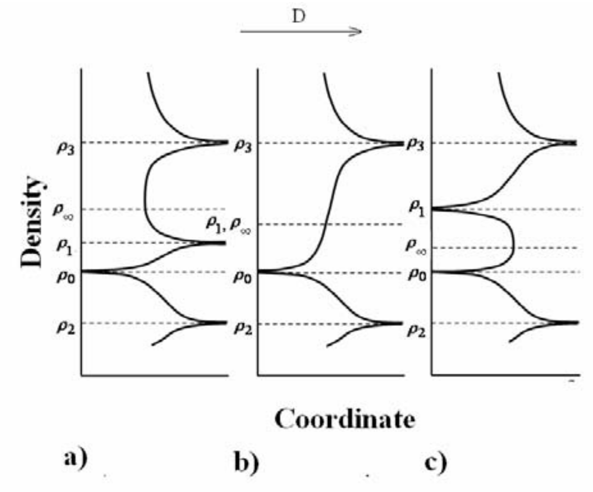

Figure 3. Integral curves of Eq. (2) in field 2. Stationary states $\rho_i, i = 1,4$ correspond to $A(\rho) = 0$, the inflection point ρ_∞ corresponds to $B(\rho) = 0$. a) $D < D_{cr1}$ b) $D = D_{cr1}$ c) $D > D_{cr1}$

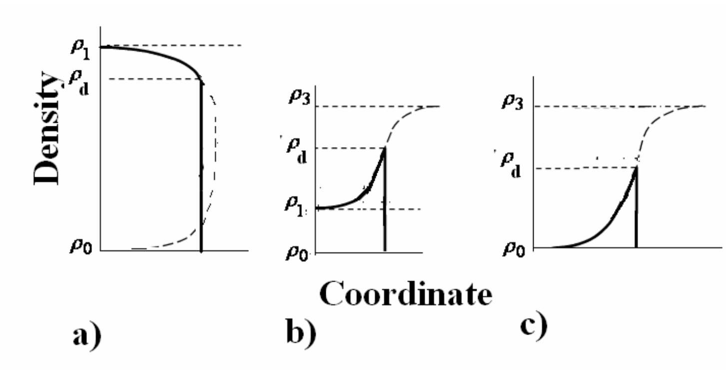

Figure 4. Shock wave structures in field 2. a) $D>D_{cr2}$; b) $D_{cr1}<D<D_{cr2}$; c) $D=D_{cr1}$ (Self-sustained pulse[21,22])

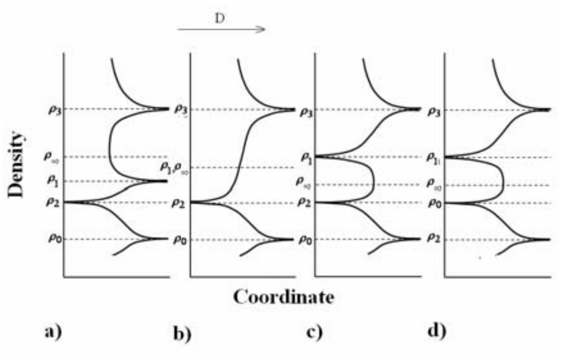

Figure 5. Integral curves of Eq. (2) in field 3. a) $D < D_{cr1}$; b) $D = D_{cr1}$; c) $D_{cr1} < D < c_0$;d) $D > c_0$

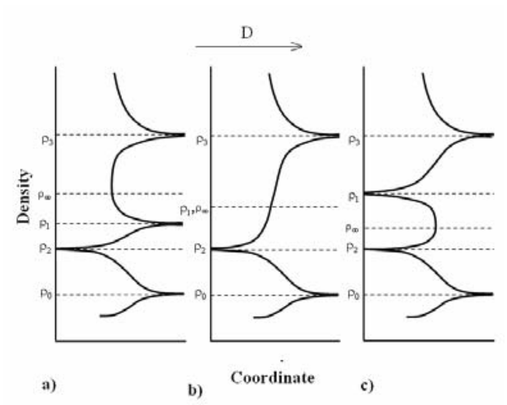

Figure 6. Integral curves of Eq. (2) in field 4,5. a) $D < D_{cr1}$;b) $D = D_{cr1}$; c) $D > D_{cr1}$

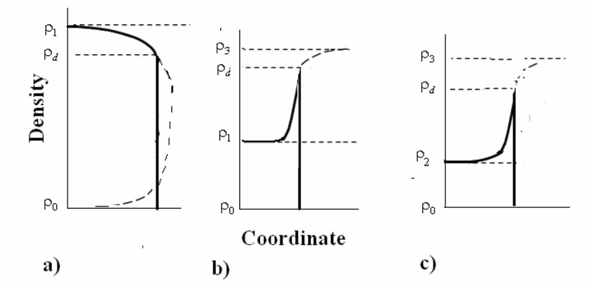

Figure 7. Shock wave structures in field 3-5. a) $D > D_{cr2}$; b) $D_{cr1} < D < D_{cr2}$; c) $D = D_{cr1}$ (Self-sustained wave with non-zero asymptote)

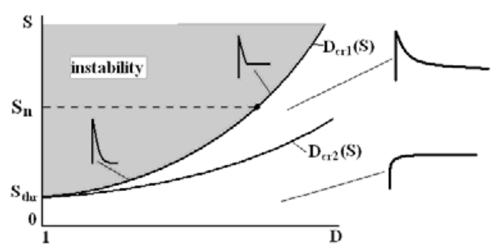

Figure 8. S-D bifurcation diagram

All results can be presented in the bifurcation diagram (Figure 8). Here, the implicit forms of boundaries D_{cr1}, D_{cr2} are

$$S = \frac{\dfrac{\tilde{T}_k}{\exp\{\frac{\tilde{T}_k}{\tilde{T}_1}\}-1} - \dfrac{\tilde{T}_k}{\exp\{\tilde{T}_k\}-1} + \dfrac{\gamma_\infty}{2(\gamma_\infty{}^2-1)}\dfrac{(\tilde{D}_{cr1}^2-1)^2}{\tilde{D}_{cr1}^2}}{1-\dfrac{\exp\{\frac{\tilde{b}}{\sqrt[3]{\tilde{T}_1}}-\tilde{b}\}}{\tilde{D}_{cr1}}},$$

$$S = \frac{\dfrac{\tilde{T}_k}{\exp\{\frac{\tilde{T}_k}{\tilde{T}_2}\}-1} - \dfrac{\tilde{T}_k}{\exp\{\tilde{T}_k\}-1}}{1-\dfrac{\exp\{\frac{\tilde{b}}{\sqrt[3]{\tilde{T}_2}}-\tilde{b}\}[(\gamma_\infty-1)\tilde{D}_{cr2}^2+2]}{\sqrt{\tilde{T}_2}(\gamma_\infty+1)\tilde{D}_{cr2}^2}},$$

$$\tilde{T}_1 = [\frac{1+\gamma_\infty \tilde{D}_{cr1}^2}{(\gamma_\infty+1)D_{cr1}}]^2 ; \tilde{T}_2 = \frac{[(\gamma_\infty-1)\tilde{D}_{cr2}^2+2][2\gamma_\infty \tilde{D}_{cr2}^2-(\gamma_\infty-1)]}{(\gamma_\infty+1)^2\tilde{D}_{cr2}^2},$$

$$\tilde{D} = \frac{D}{c_\infty}, \tilde{T}_k = \frac{T_k}{T_0}, \tilde{b} = \frac{b}{\sqrt[3]{T_0}}.$$

The obtained shock wave structures can be easy explained with help of shock adiabats. Here, stationary states $\rho_i = 1/V_i, i = 1,4$ correspond to points of intersection of the chord drawn from initial point $0(P_0,V_0)$ and the equilibrium adiabat, state ρ_d corresponds to point of intersection of this chord with the frozen adiabat. As an example, we consider the nonequilibrium field 2 ($S_{thr} < S < S_n$), where dispersion is negative. The inclination of the chord drawn from initial point $0(P_0,V_0)$ to a cross-point of adiabatic curves (P_{cr2},V_{cr2}) defines the critical velocity of a shock wave

$$D_{cr2} = V_0\sqrt{\frac{P_{cr2}-P_0}{V_0-V_{cr2}}}.$$

For the shock wave velocity $D > D_{cr2}$, a corresponding chord inclination is greater (Figure 9 a, chord 0d1). Here, the shock wave structure is typical of relaxing media with the positive dispersion, because the point of intersection between the chord and the frozen adiabat is to the right of the point of intersection with the equilibrium adiabat. Therefore, the medium is first rapidly compressed to the value specified by the point d of intersection between the corresponding chord and the frozen adiabat and then is gradually compressed to the final state 1 specified by the intersection of the chord with the equilibrium adiabat. The related shock wave pattern is shown in Figure 9b.

The shock wave with $D = D_{cr2}$ has step-wise form with the amplitude $P_{cr2} - P_0$. For the shock wave velocity $D_{cr1} < D < D_{cr2}$, the shock wave structure changes (Figure 9d).

In this range, the frozen adiabat is to the left of the equilibrium one (Figure 9a, chord 01d). Here, fast compression to the value d determined by the intersection of the corresponding chord with the frozen adiabat is followed by gradual expansion to the final state 1 specified by the intersection between the chord and the equilibrium adiabat.

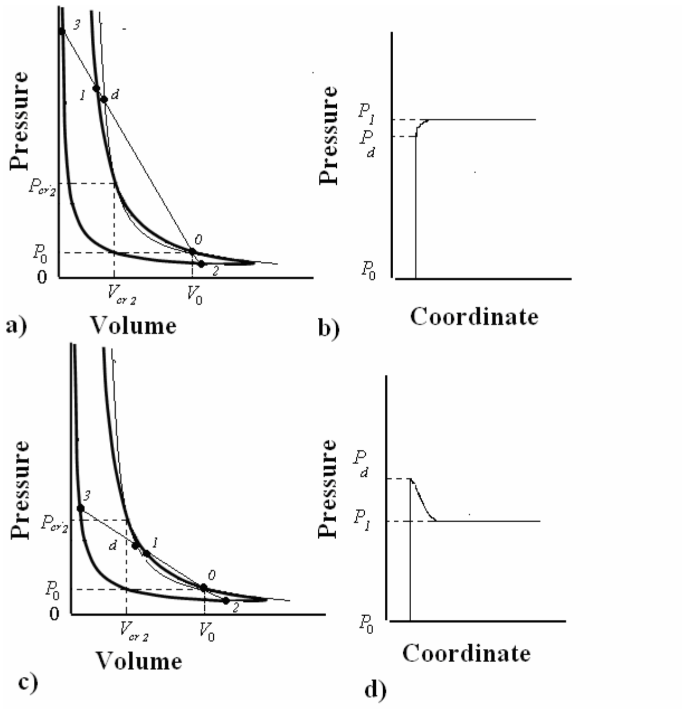

Figure 9. Shock adiabats (a,c) and shock waves (b,d) in the negative-dispersion gas. The chord 0d1 (a) corresponds to the shock wave (b) with the velocity $D > D_{cr2}$ .The chord 01d (c) corresponds to the shock wave (d) with $D_{cr1} < D < D_{cr2}$ [23]

As well known, the shock wave becomes unstable if the velocity of sound propagating behind the shock wave front is less than the shock wave velocity. This condition permits us to obtain the value of D_{cr1}. For $D = D_{cr1}$, the stationary wave has form of pulse with amplitude ρ_d [21,22].

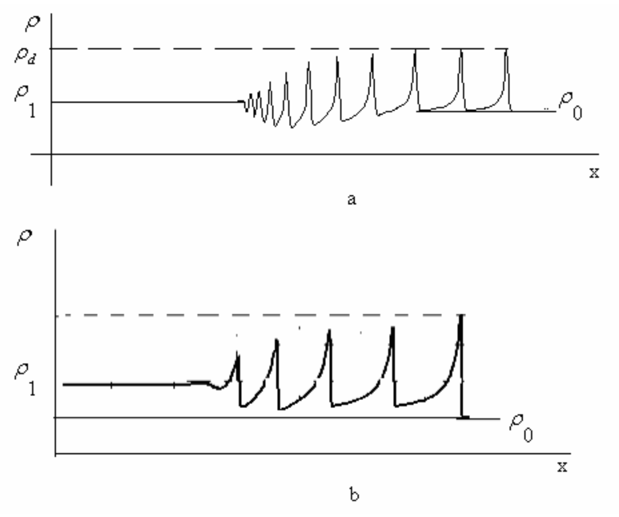

Figure 10. Autowaves, propagating with $D = D_{cr1}(S)$

For $D < D_{cr1}$, the shock wave is unstable. In field 2 ($S_{th} < S < S_n$), unstable waves with $D \le D_{cr1}$ disintegrate into sequence of solitary self-sustained (autowave) pulses with amplitude ρ_d, propagating with the boundary speed $D = D_{cr1}(S)$ (figure 10a). In the strongly nonequilibrium media (fields 3-5) for $D \le D_{cr1}$, we obtain the self-sustained wave with non-zero asymptote (figure 10b), propagating with the boundary velocity $D = D_{cr1}(S)$ as well.

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