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OPTIMAL DESIGN FOR SUPERSONIC AND TRANSONIC VELOCITIES

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Key words: optimal design of fore-bodies, bodies, shape of the plane airfoil and 3-D blade of ventilator, two-sided and boundary extremum section ,indirect and direct methods, genetic algorithms, Pareto-optimization, local linearization.

Abstract. The gas dynamics variation problems methods and results are discussed. The various design problems are considered: two-dimensional and axisymmetrical fore-bodies and closed bodies realizing a minimum of a wave drag; spatial configurations of minimum full drag; optimum by Pareto airfoils when transonic flow and different types of nozzles, realizing a maximum of thrust. The indirect and direct methods of solving the optimal design problems are based on local, semilocal and exact flow models. In the indirect approaches when obtaining the necessary optimality conditions along with «standard» methods of the optimum control theory and «the method of an uncertain control contour» is used. For the direct approaches disposal there are attracted: the Bezier spline approximation of desired contour, genetic algorithms (GA) and local linearization.

INTRODUCTION

The first variation problem of aerodynamics is the design problem of an axisymmetrical forebody with given aspect ratio realizing a minimum of drag (Newton's problem – NP). It was solved still by Newton [1] with using of the «local» formula, proposed by him, (Newton's formula – NF) for determining the pressure p on a forebody's surface. Later both mathematicians, and mechanics returned to NP, that stimulated the development of optimum control theory. The important element of the NP solution is the forward base, which has appeared as a boundary extremum segment (BES) because of the limitation of the forebody's length. In spite of the fact that NP was the first, or at least one of the first variation problem, the boundary extremum sites in calculus of variations classical courses, as a rule, were not considered. For this reason, the aerodynamicians that have approached to NP in 50-th years of the XX century and have knew only two-sided extremales (TSE), satisfying to Euler equation, not at once have managed to find the right solution or even to understand the Newton's outcomes. The questions with NP were arising either earlier. So, Legendre, allegedly, has given the recipe of improving the Newton's solution. The analysis of this recipe has shown, however, that the problem's formulation is needed to add the requirements of the NF applicability. As a result the forward base became the BES both by the length, and by the boundary of applicability the NF (A. Kraiko, 1963). The serious hardship have arisen when designing in NF frameworks of a forebody, which is optimum for a given volume (instead of length), and also when given the both overall dimensions and volume. In these problems the new BES occur: a forward base of a given cylindrical forebody with the outstanding from it the sharp or blunt pin, and the end cylindrical section of as much as possible permissible radius (see below).

In 60-70-th years of XX century G. Chernyi, A. Gonor, A. Miele, G. Saaris and D. Hull have solved the first variation problems about optimum spatial bodies in NF frameworks and also in NF with the local laws of friction. The drag of the forebodies, designed by them, with star-shaped cross sections has received noticeably less than that of the equivalent by the length and volume circular cones.

Along with the local approaches based on NF and its generalizations, and also on the linear theory of two-dimensional supersonic flows, the approaches were developed from 1950 using equations of two-dimensional and axisymmetrical supersonic flow of ideal (inviscid and not heat-conducting) gas (the Euler equations). Here the greatest promoting was reached in solving of the design problem for a supersonic part of Laval nozzles realizing a maximum of thrust (K. Guderley, E. Hantsch, Yu. Shmyglevskii, L. Sternin, G.V.R. Rao, A. Kraiko, J. Armitage, V. Borisov, I. Michailov, V. Butov, I. Vasenin, A. Osipov, N. Tillyayeva) and afterbodies of a minimum wave drag. For forebodies streamlined with an attached shock wave, the particular solutions corresponding to (in the special cases) two-dimensional straight-line generatrix (G. Chernyi, 1950) and smooth contours of bodies of revolution with a duct (Yu. Shmyglevskii, 1957) were found. The solving of listed and other problems has required the development of body of mathematics that is of common interest for the optimum control theory of distributed parameter systems. It includes the method of a control contour – CCM (A. Nikolskii, 1950; K. Guderley and E. Hantsch, 1955); an uncertain control contour method – UCCM (A. Kraiko, 1979) as the validation of the G.V.R. Rao's approach (1958); a variation in a characteristic strip (G. Chernyi, 1950), a common statement of a Lagrangian multiplicities method – LMM (K. Guderley and J. Armitage, 1962); the allowing of the multipliers breaks along the characteristics in LMM (A. Kraiko, 1964); a variation of inclination break streamlined with forming of the centered waves of rarefaction (A. Kraiko, 1966). Having applied the LMM to designing of optimum forebodies streamlined with an attached shock wave, A. Shipilin (1966) with the help of extremely complicate numerical algorithm has constructed 7 two-dimensional and 14 axisymmetrical (for ducted bodies) optimal generatrix with an internal convex inclination break.

The impression about the solved in 1950-70-th variation problems of gas dynamics, including approximate (local models of streamlining and one-dimensional model of flow in channels) and exact (the Euler equations) statements and developed for this purpose methods, give the monographies [2-4] and review [5].

SOME RESULTS OF LAST 10-15 YEARS

For the last 10-15 years the noticeable advance is reached in solving of the design problem of optimal aerodynamic shapes. To confirm it we shall put the outcomes obtained in author's collective.

Let's begin from two-dimensional forebodies, which realize a minimum of a wave drag within the framework of the Euler equations when given the overall dimensions and Mach number $M_\infty > 1$ of entering airflow. For each M_∞ there are no more than three values of relative thickness (ratio of height to length) $\tau > 0$, for which the solution of this problem gives a straight-line generatrix (wedge). For others τ within the framework of ideal gas the optimal contour has the infinite number of inclination breaks and points of their concentration [3]. But only one of them the main inclination break however noticeably influences on a wave-drag coefficient C_x. As it was already noted, A. Shipilin with the help of LMM and rather complicate numerical algorithm has designed several of such contours. In [6] the analytical approach is advanced which permits practically instantaneously to build close to optimal two-dimensional forebodies with one internal inclination break for all M_∞ and τ, that keep the streamlining with attached head shock and $M \geq 1$ in all flow. The executed calculations have allowed determine the contribution of the inclination break to value C_x, which has appeared minor in all cases.

Obtained in [6] the information on the contribution of internal inclination breaks to value C_x stimulated the development of effective and rather precise methods of designing close to optimal forebodies and profiles generatrices. When designing profiles, along with the length of a chord, accepted for a linear scale, the square of longitudinal section $2F$, divided by square of a chord length and lift coefficient C_y are fixed. Because of the length limitation the optimal profile contours can contain a back base (BB) – one more of a boundary extremum sites [7-9]. On BB close to constant the base pressure $p^+ > 0$ acts. Within the framework of the linear theory for supersonic flows a capability of BB appearance was at first denoted by D. Chapman (1953). Enough, when moderate supersonic velocities the requirements of BB existence are executed only for rather thick profiles (for major F). When $M_\infty >> 1$ the necessity of BB appearance arises for rather small F and even when $p^+ = 0$. In such cases C_x of optimal profiles with BB occurs much less, than C_x of pseudo-optimal profiles with the sharp trailing edge.

For optimum airfoil design the modification of a «shock-expansion method» (SEM) is made [8]. According to SEM the pressure in an arbitrary point of the airfoil, streamlined with a joined shock, is determined by angles ϑ and ϑ_i of tangent declination in considered and in forward points of a contour required. When solving the variation problems it reduces in physically senseless outcome – the finite change of C_x when only forward point contour inclination modification. In offered modified SEM [8] – MSEM the angle ϑ_i is exchanged with a mean angle of a velocity inclination on a site of a head shock from point i up to its such point, for which going out from it C^--characteristic comes to the end point of a streamlined by a supersonic flow body site. The investigations executed have shown the high accuracy of MSEM. For optimum airfoils designed the values of C_x, determined by the integration of the Euler equations and by incomparably more prime MSEM, as a rule, differ on a part of percent.

M_∞	p^+	$C_x \cdot 10^2$	ΔC_x (%)	$\tau \approx y_f$
6	0	1.19	149	0.148
12		0.78	223	0.157
6	p_∞	0.89	233	0.154
12		0.71	258	0.158
6	$2p_\infty$	0.58	412	0.161
12		0.64	303	0.160

Table 1. Wave drag coefficients for different p^+

The effect of BB introduction we shall show by the example of thick symmetrical profiles ($F = 0.096$), streamlined under a zero angle of attack by perfect gas flow with adiabatic ratio $\kappa = 1.4$ and $M_\infty = 6$ and 12. The shape and C_x – value of optimal contours with BB depend on p^+ – value, which was taken equal 0, p_∞ and $2p_\infty$. Determined by integration of the Euler equations values C_x of profiles designed in the base of MSEM, are assembled in the table 1 and on fig. 1. Along with C_x the balance ΔC_x over pseudo-optimum profiles with a sharp trailing edge are presented here, and also relative widths τ and practically equal to them ordinates y_f of BB. For wedges with same F the value ΔC_x corresponds to 30-35 %. For $M_\infty = 6$ in fig. 1 the optimal profiles designed in the base of MSEM are shown by the solid curves, and those designed by NF when $p^+ = p_\infty$ are drawn by the dashed lines (x is related to the body length, η is the ordinate, related to the semi-thickness of pseudo-optimum body without BB). When $M_\infty \gg 1$ thin high-lift profiles with BB are also noticeably benefit in comparison with sharp-ended those, though it's not so significant, as in considered examples [9].

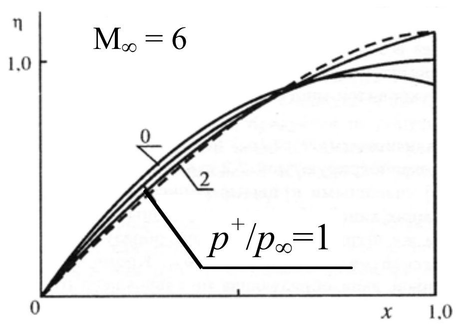


Figure 1. Optimal contours for different p^+

As it was already noted, in 1960-70 within the framework of the local laws of drag the first optimal spatial bodies were designed. However along with the major number of assumptions (about a delicacy, homoteticity or conicity of unknown construction) the obvious defect of all obtained then solutions was the impossibility of joining the star-shaped forebody with axisymmetrical base. A first step to correct situation was made by G. Chernnyi and V. Levin with the colleagues (1979). They designed the required surface with lined surfaces stretched on a forward cross with $N \geq 2$ rays and on a circle. In 2000-2002 the grand break in this way has made G. Yakunina [10-12]. She advances a method, unexpectedly simple and not requiring the listed above simplifications, of designing the optimal space configurations, including a star-shaped conical, not conical of aircraft type and spatial forebodies of given length with the circular base (Fig. 2).

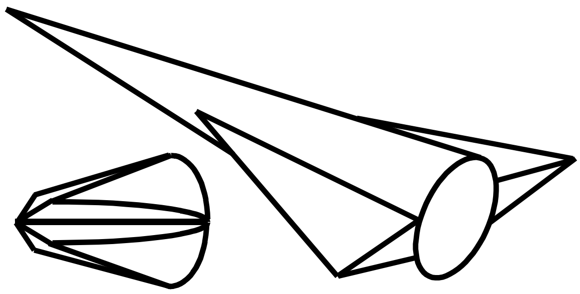


Figure 2. Spatial configurations, optimal in approximation of drag local model

The results of [6-12] have made the base of the monography [13]. The latter also includes advanced earlier (A. Kraiko, V. Shelomovskii, 1984) method of designing the smooth generatrices for body of revolutions with a duct, close to optimal in the Euler equations approximation. Basing upon this method, A. Kraiko with the colleagues [14] in the same approximation have designed close to optimal axisymmetrical forebodies of arbitrary fineness ratio. The designed bodies have a forward base. For small fineness ratio the obtained solution is precise.

Let's return to optimization in terms of NF and other local interaction laws (LIL). In the NF approximation for thin bodies the problem of optimum fore-body is considered in [15] when given length L, the base radius R and volume W (more exact, $l = L/R$ and $C_\Omega = \Omega/(\pi R^2 L)$). The solution has appeared not full, for obtained in [15] equations and the requirements permit to design optimum fore-bodies only when $0.25 \le C_\Omega \le 0.5$ – in a quarter of a full interval of volume coefficient values ($0 \le C_\Omega \le 1$).

In [16] there are designed the axisymmetrical fore-bodies realizing a minimum of wave drag when fixed overall dimensions and a volume in approximation of NF. For great volumes the desined forebodies can include alongside with an already mentioned forward base a horizontal segment of maximum permissible radius, which is another site of boundary extremum. When small volumes the forebodies, found in NF frameworks, have the shape of a pin running out of a base which is the left boundary of a given body of revolution. The determining of wave drags for the designed forebodies in an approximation of supersonic gas dynamic full equations (Euler equations) has reduced to modification of the initial variation problem formulation. According to new formulation the design of fore-bodies with a volume, that is smaller than some one (which is gained for the given aspect ratio without volume setting), appears senseless.

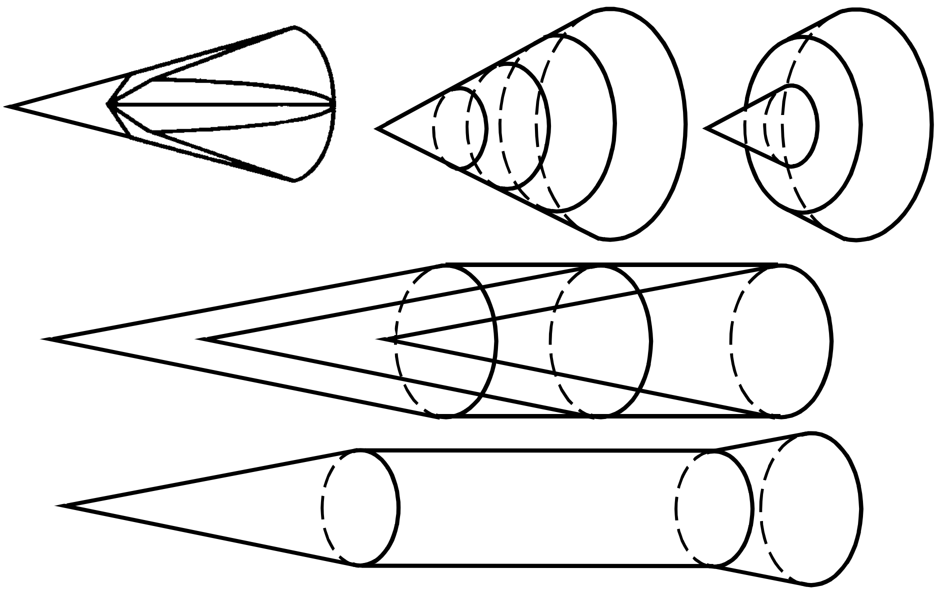


Figure 5. Optimal bodies when given the base square S_0 and ratio S_0/S

One of the problems, solved in a LIL approximation, is the design of a minimum drag body when fixed squares of the base S_0 and windward section S. For almost half-centuries it was wrongly considered (see, for example,[17]), that the solution does not depend on a LIL sort and when circular base the optimum body is a cone with apex angle being determined by value of ratio $S_0/S \le 1$. Recently it is established [18], that it is not true. Far not full variety of optimum bodies, being obtained in this problem, is shown in fig. 5.

Until recently the main achievements in optimal nozzles design are concerned of designing the supersonic part of a Laval nozzle. For last years the frame of problems, solved in this way, has essentially extended. The designing of self-controlled plug nozzles is made [19-21] and of expansion-deflection nozzles [22, 23]. The methods of definition their performances for «off-design» conditions are advanced (fig. 6, $\pi = p_0/p_e$, p_0 is the total pressure, p_e is the external one). When designing the supersonic parts of such nozzles with major turn angles of flow the presence of an initial isobaric site is essential. The compression waves, going from it, form a shock, well outstanding in fig. 6.

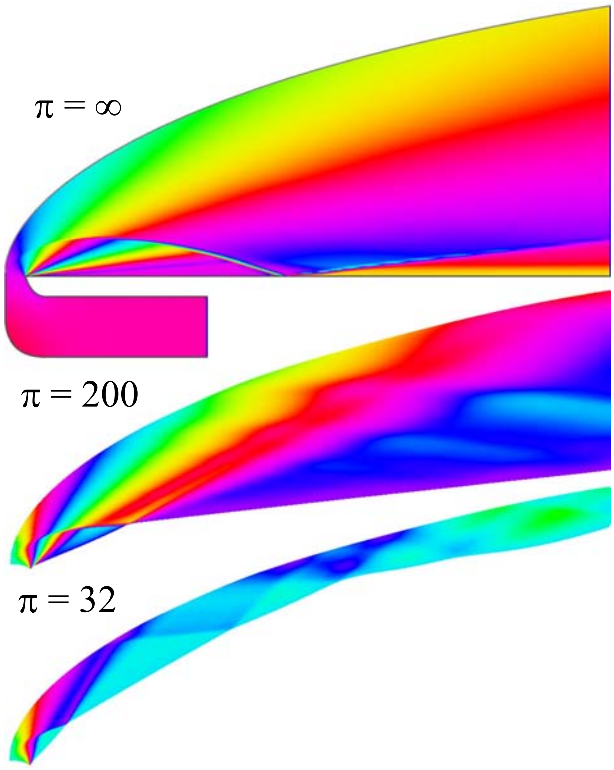


Figure 6. Flow in optimal expansion-deflection nozzle when design and on-design conditions

In [19-23] the design of optimal contours is made in approximation of ideal gas, but when definition the thrust losses (especially for off-design conditions with separations) it is taken into account the gas nonideality, and among them, on the base of full Reynolds equations (RE), closed by modern models of turbulence. Similarly in [24] all Laval nozzle (including a subsonic part) is designed in an ideal approximation, and features of flow and the losses are determined by integration of RE. In this case the account of gas nonideality is important because for an optimal contour of all nozzle the role of a subsonic part is implemented by abrupt diversion.

As shown in [24], when fixed length of all nozzle the advantage of nozzles with abrupt diversion in comparison with the smooth entry nozzles becomes when account of viscosity even more considerable, than in an inviscid approximation. The indicated advantage is saved even for nozzles, designed without accounting of the strong non-uniformity of entry flow. The example of flow in such a nozzle is presented in fig. 7. Despite of an intensive hanging shock, formed because of improper supersonic part of nozzle design, it provides the better thrust, than smooth entry nozzle with the same overall length.

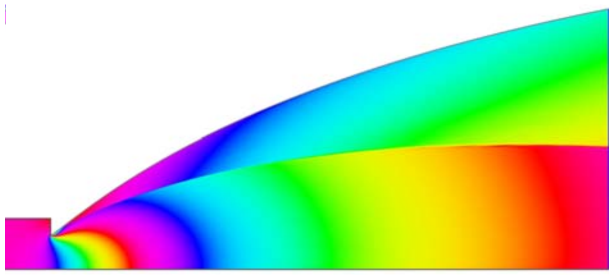


Figure 7. Flow in the nozzle with abrupt diversion when nonoptimal contour of its supersonic part

Problems solved in [25, 26], are connected to designing of asymmetrical two-dimensional nozzles. In [25] the asymmetrical nozzles of maximum thrust are designed when given overall dimensions and moment, and in [26] – asymmetrical nozzles realizing the maximum of thrust impulse module for the device with a rotated valve. In the second case the minimal cross-section of the nozzle and parameters in the combustion chamber periodically vary by the time, and the combustion occurs when enclosed nozzle. The same approach is applied to design the nozzles of pulse detonation engines.

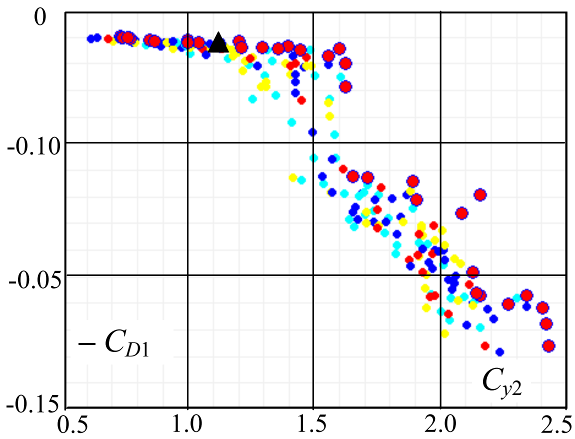


Figure 8. Pareto optimization of airfoil for cruise conditions ($M_{\infty1} = 0.8$) and when take-off ($M_{\infty2} = 0.3$)

The discussed above outcomes are obtained with the help of "indirect" methods of the optimum control theory. During last years the "direct" methods were also intensively developed. For problems of a multicriteria optimization, in which the optimization of one performance is carried out when acceptable level of others ("Pareto optimization") the genetic algorithms – GA are rather perspective.

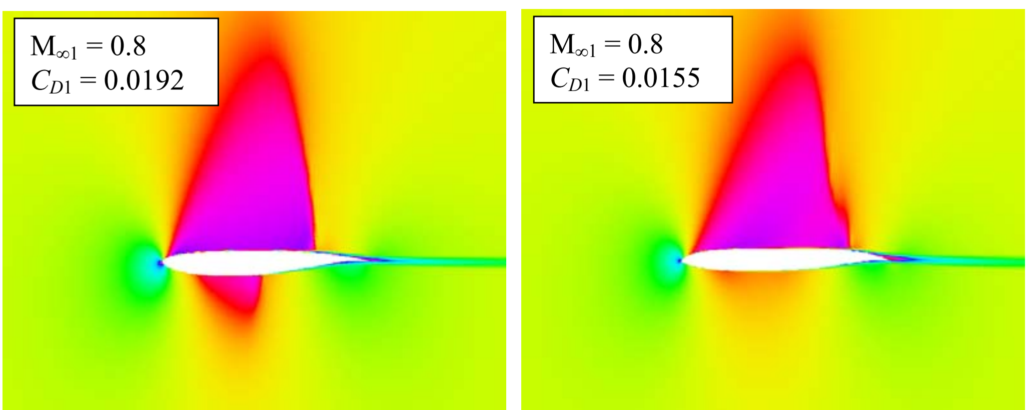


Figure 9. Flow of airfoils, designed by method [27] (on the left) and

by GA (on the right) when $M_{\infty 1} = 0.8$

The designed with the help of GA optimal by Pareto wing profile of fixed longitudinal square $F/l^2 = 0.0802$ (l is the chord length) is below presented. When given lift coefficient $C_{y1} = 0.55$ for cruise regime ($M_{\infty 1} = 0.8$) the designed airfoil should have the low drag coefficient C_{D1} for this regime, and when take-off ($M_{\infty 2} = 0.3$) ensure a maximum of C_{y2}.Therefore the Pareto front for optimized airfoils is designed in a plane C_{y2}, $-C_{D1}$. The shapes of wing profiles were set with Bezier-splines, and the flow field was calculated by integration of Reynolds equations, closed by the turbulence model of Spalart-Allmaras.

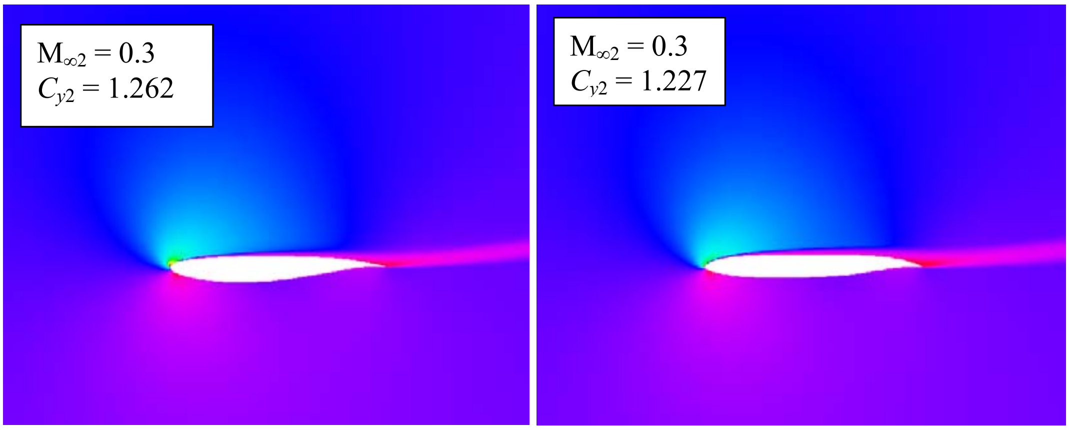


Figure 10. Flow of airfoils, designed by method [27] (on the left) and
by GA (on the right) when $M_{\infty 1} = 0.3$

In fig. 9 the results of Pareto-optimization («the fronts of Pareto » after some number of iterations) are shown. The size of a population in GA was numbered for 80 persons. The airfoil shape was set by Bezier splines, containing 11 degree of freedoms. The iterations number in GA was about 200. The airfoils performances, found as a result of Pareto-optimization with a coarse grid, then were updated by calculations with a fine grid.

In fig. 9 and 10 the Mach number fields, implemented when flow with $M_{\infty 1} = 0.8$ (fig. 9) and with $M_{\infty 2} = 0.3$ (fig. 10) are shown. Their lower pictures represent the flow of airfoil, being correspond the black triangle in fig. 8. The airfoil presented in upper pictures is designed with the help of the developed in [27] method of correction. In [27] the approach of H. Sobieczky for design of supercritical airfoils is realized within the framework of a «time evolution till quieting» procedure and method of characteristics with introduction at intermediate stage of «artificial» gas. Having only slightly (2.9 %) less Cy when take-of, the airfoil, designed by a direct method with use of GA, has for cruise condition ($M_{\infty 1} = 0.8$) 24 % smaller drag, than airfoil corrected by the method [27].

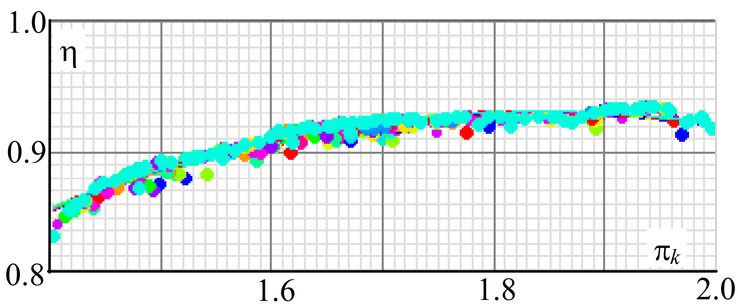
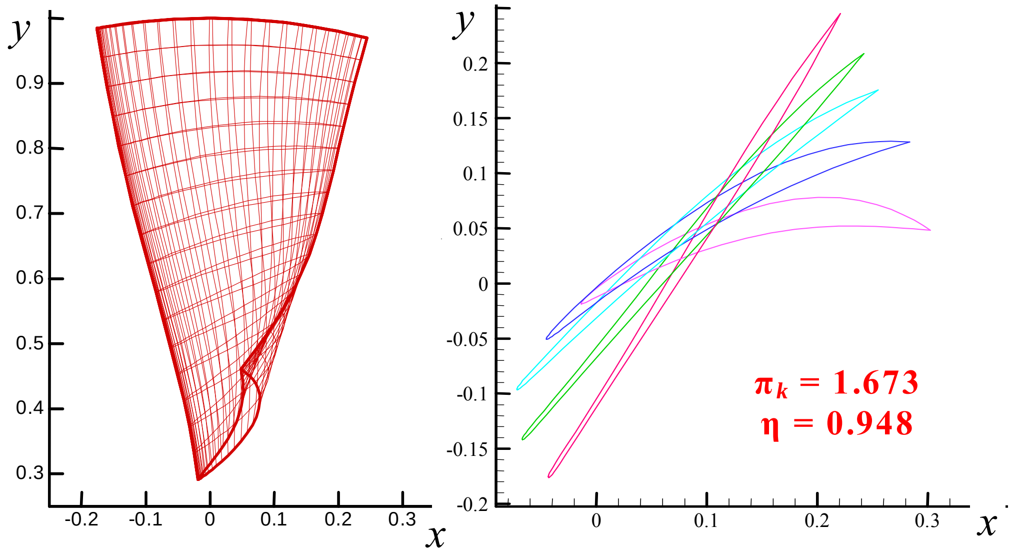


Figure 11. The Pareto-front when optimization of fan’s blade

Fig. 11 and 12 illustrate the design with using of GA the Pareto-optimal 3D-blades of fan's rotor in the plane of two characteristics: compression ratio π and efficiency coefficient η. The blades shapes were determined by Bezier splines, the flow was computed by the integration of 3D-equations of Reunolds, closed by the Secundov's differential turbulent model «ν_t-90», with parallelization of calculations for 20-30 processors. In fig.11 the fronts of Pareto are presented, determined with a coarse grid, in fig. 12 – one of optimal blades and its profiles at the bush, on a rim and in intermediate sections (π_k and η are recalculated with a fine grid).



Фиг. 12. Optimal blade: the view from below the flow and profiles in the different sections

The main disadvantage of GA, as well as classic direct methods, is the necessity of repeated flow calculation of optimized object for determining the direction of motion («the steepest descent») in N-size space of parameters d_n, defining its shape. In these methods at each step it is required to execute N calculations. One of tools to reduce their number up to unity is the coefficients of sensitivity method (CSM), with coefficients, defined from the solution of some "conjugate" problem (CP). When rigorous realization it is equivalent to CP for Lagrangian multiplicities in LMM. As a rule, CP of LMM are so complicated, that in CSM their simplified approximated analogs are used. Exception are the design problems of smooth surfaces streamlined with M ≥ 1, though either for them in CP of LMM there can be mentioned above lines of multiplier break, complicating the solution. Besides CSM gives only direction of a modification d_n, but not the value of their increments. S. Takovitskii has offered the approach [28-30], more effective, than CSM for problems of this class. Idea of a method [28-30], usable for design of 2D and 3D-optimal configurations, we shall explain by the example of designing the axisymmetrical fore-body, that realizes the minimum wave drag when given L, the base radius R and volume Ω, more exact, when given the aspect ratio $l = L/R$ and the volume ratio $C_\Omega = \Omega/(\pi L R^2) \leq 1$.

The results of this problem solving in terms of NF [16] are presented in fig. 13. The end points of the curves $C_x = C_x\,(C_\Omega,\,l)$ correspond to the minimum and maximum possible volumes ($C_\Omega = 0$ and 1). For these, according to NF, $C_x = 1$. For each l the circles show the value of C_x of the nose shape of Newton's problem, which is a minimum for this l. Between the circles and triangles the optimum contours have a front face and a gently sloping segment of a bilateral extremum. The triangles correspond to the limitingly thick fore-bodies of this kind with a horizontal tangent at the right end the point. At the right of these the contours consisting of the front face, a gently sloping part and a rear cylinder $r \equiv R$ are optimum.

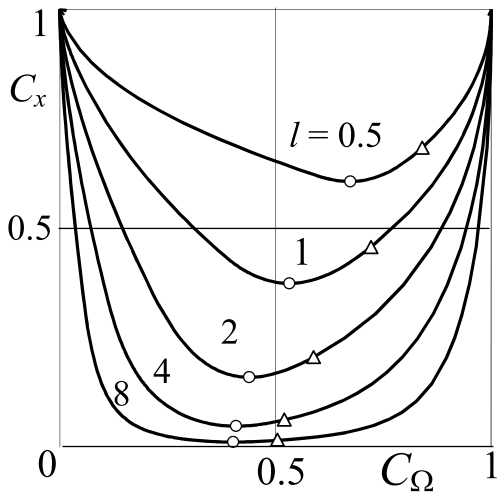


Figure 13. The results of optimum fore-body design using the NF

The values of C_x calculated in [16] by integration of the equations of the perfect flow for the nose shapes, constructed there using NF , confirmed the advantages of convex configurations. In contrast to this, C_x of nose shapes with concave parts **(**to the left of the small circles in fig 13) and all the more with base faces, as a rule,exceeded C_x of the equivalent cones. At the sometime, in the examples, where the advantage of such nose shapes preserved, it was considerably less than that obtained using NF. One of the consequences of these results is a change in the formulation of the variational problem. In practice, the specification of the volume of the fore-body is due to the arrangement in it of a useful load of fixed volume Ω_f. For this volume the nose shape must satisfy the inequality $\Omega \geq \Omega_f$. If Ω^0 corresponds to the solution of NP with a free volume $\Omega^0 \geq \Omega_f$, then precisely this with $C_x = C_x^0$, which is a minimum for specified l, gives a solution of the problem. According to fig. 13 in this case a considerable reduction in the drag is possible (when $l \geq 1$ – several fold). From a mathematical point of view, such a nose shape can be regarded as a hollow fairing with walls of zero thickness with the useful volume Ω_f inside. Consequently, nose shapes that are optimum when using NF with $C_\Omega < C_x^0$, corresponding to the parts of the curves to the left of the small circles in fig 13 are of no interest.

Taking the above into account henceforth the coordinates of the contours of the optimum fore-bodies and their volume coefficients will satisfy the constraints

$$0 \leq x \leq L,\ \ 0 \leq r \leq R,\ \ C_\Omega^0 \leq C_\Omega = \frac{1}{LR^2}\int_0^L r^2 dx \leq 1 \qquad (1)$$

The coefficients C_Ω^0 and C_x^0 for specified l, M_∞ and the specific heat ratio κ correspond to the solution of NP using Euler equations, which describe the ideal flow, gas.

The basis of the method is the representation of the increment of the drag by a quadratic form. When constructing it, local linearization is employed, which gives a relation between the increments of the pressure and the change in the orientation of small elements of the contour or surface in the flow. The conditions for a minimum of the quadratic form lead to equations that ensure a close-to-quadratic rate of convergence to the optimum.

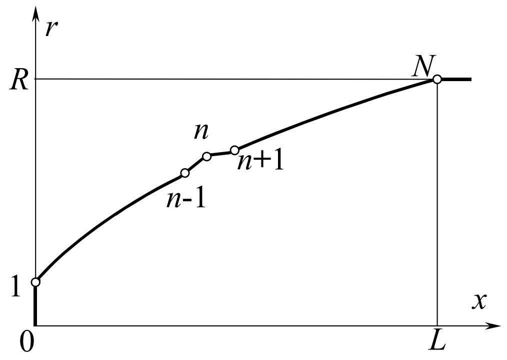


Figure. 14. The varying of the forebody shape in the method of “local linearization”

We will begin the description of the method employed with the example of a nose shape without a cylindrical part. We will take as the controls the values r_n of the radii of the ends of the N-1 segments of the gently sloping part of the nose shape with fixed x_n (n = 1, …, N). From the coordinates of the front face r_1 and $x_1 = 0$ and of the base $r_N = R$ and $x_N = L$ only r_1 is a control. The mean parameters on the segments will be assigned semi-integer subscripts, for example (fig. 14) $h_{n-1/2} = x_n - x_{n-1}$. In accordance with this, the formulae for χ – the integral of pressure forces, acting on the fore-body, and its volume $\Omega \geq \Omega^0$ apart from positive factors which will be unimportant later, take the form

$$\chi = \int_0^{r_t} prdr + \int_0^L prr'dx = \int_0^{r_t} (p - p_\infty)rdr + p_\infty \frac{r_t^2}{2} + \int_0^L prr'dx = \frac{Kr_t^2}{2} + \int_0^L prr'dx \approx$$

$$\approx \frac{Kr_1^2}{2} + \sum_{n=1}^{N-1} (prr'h)_{n+1/2}, \quad \Omega = \int_0^L r^2 dx \approx \frac{1}{3}\sum_{n=1}^{N-1} (r_n^2 + r_n r_{n+1} + r_{n+1}^2) h_{n+1/2} \qquad (2)$$

$$K = \rho_\infty V_\infty^2 \left(c_{x0} + \frac{1}{\kappa \mathrm{M}_\infty^2} \right), \quad c_{x0} = c_{x0}(\mathrm{M}_\infty, \kappa) = \frac{1}{r_t^2} \int_0^{r_t} \frac{p - p_\infty}{\rho_\infty V_\infty^2} dr^2$$

Here $r_1 = r_t$ is the radius of the front face while c_{x0} is its wave drag coefficient. For comparatively slight kinks at the point t the quantity c_{x0} is independent of the shape of the gently sloping part. For all examples in [30] this condition is satisfied.

A change in the radii r_n yields changes in the angles of inclination of the segments ϑ and the pressure p, acting on them. We will assume, that the relation between the increments Δp and $\Delta\vartheta$ is the same as for local linearization of the flow equations with respect to the parameters of the initial supersonic flow (but different from the free stream) on the same segments. In this approximation, it is reduced to the formular for a simple wave(μ is the Mach angle)

$$\Delta p \approx \rho V^2 \mathrm{tg}\mu \Delta\vartheta \approx A \Delta \mathrm{tg}\vartheta, \quad A = \rho V^2 \mathrm{tg}\mu \cos^2 \vartheta$$

In accordance with this

$$\mathrm{tg}\vartheta_{n-1/2} = \frac{r_n - r_{n-1}}{h_{n-1/2}}, \quad \Delta(\mathrm{tg}\vartheta_{n-1/2}) = \frac{\Delta r_n - \Delta r_{n-1}}{h_{n-1/2}}$$
$$\Delta p_{n-1/2} \approx A_{n-1/2}\frac{\Delta r_n - \Delta r_{n-1}}{h_{n-1/2}}, \quad \Delta p_{n+1/2} \approx A_{n+1/2}\frac{\Delta r_{n+1} - \Delta r_n}{h_{n+1/2}} \tag{3}$$

According to relations (2) and (3), the increments $\Delta\chi$ and $\Delta\Omega$ apart from squares of Δr_n are given by the expressions

$$\Delta\chi = \left[Kr_1 - \left(Arr' + rp - pr'\frac{h}{2}\right)_{3/2}\right]\Delta r_1 + \left[\frac{K}{2} - \left(\frac{Ar'+p}{2} - \frac{A}{h}r\right)_{3/2}\right](\Delta r_1)^2 -$$
$$-2\left(\frac{A}{h}r\right)_{3/2}\Delta r_1\Delta r_2 + \sum_{n=2}^{N-1}\left\{\left[\left(Arr' + rp + pr'\frac{h}{2}\right)_{n-1/2} - \left(Arr' + rp - pr'\frac{h}{2}\right)_{n+1/2}\right]\Delta r_n + \right.$$
$$\left. + \left[\left(\frac{Ar'+p}{2} + \frac{A}{h}r\right)_{n-1/2} - \left(\frac{Ar'+p}{2} - \frac{A}{h}r\right)_{n+1/2}\right](\Delta r_n)^2 - 2\left(\frac{A}{h}r\right)_{n+1/2}\Delta r_n\Delta r_{n+1}\right\} \tag{4}$$
$$3\Delta\Omega = (2r_1 + r_2)h_{3/2}\Delta r_1 + h_{3/2}(\Delta r_1)^2 + h_{3/2}\Delta r_1\Delta r_2 + \sum_{n=2}^{N-1}\{[(r_{n-1} + 2r_n)h_{n-1/2} +$$
$$+(2r_n + r_{n+1})h_{n+1/2}]\Delta r_n + (h_{n-1/2} + h_{n+1/2})(\Delta r_n)^2 + h_{n+1/2}\Delta r_n\Delta r_{n+1}\}, \quad \Delta r_N = 0$$

To obtain the necessary conditions for a minimum of χ for fixed Ω, setting up the Lagrange functional $I = \chi + 3\lambda\Omega$ with undetermined constant multiplier λ, we equate the derivatives of ΔI with respect to Δr_n to zero with $n = 1, 2, \ldots, N$-1. Together with the condition of a fixed volume, $\Delta\Omega = 0$, this gives a system of N equations for determining all the Δr_n and λ

$$(2r_1 + r_2 + \Delta r_1 + \Delta r_2)h_{3/2}\Delta r_1 + \sum_{n=2}^{N-1}[(r_{n-1} + 2r_n + \Delta r_n)h_{n-1/2} +$$
$$+(2r_n + r_{n+1} + \Delta r_n + \Delta r_{n+1})h_{n+1/2}]\Delta r_n = 0,$$
$$Kr_1 - B^-_{3/2} + \lambda(2r_1 + r_2 + 2\Delta r_1 + \Delta r_2)h_{3/2} + (K - C^-_{3/2})\Delta r_1 - D_{3/2}\Delta r_2 = 0,$$
$$B^+_{3/2} - B^-_{5/2} + \lambda[(r_1 + 2r_2 + \Delta r_1 + 2\Delta r_2)h_{3/2} + (2r_2 + r_3 + 2\Delta r_2 + \Delta r_3)h_{5/2}] - \tag{5}$$
$$-D_{3/2}\Delta r_1 + (C^+_{3/2} - C^-_{5/2})\Delta r_2 - D_{5/2}\Delta r_3 = 0,$$
$$B^+_{n-1/2} - B^-_{n+1/2} + \lambda[(r_{n-1} + 2r_n + \Delta r_{n-1} + 2\Delta r_n)h_{n-1/2} + (2r_n + r_{n+1} + 2\Delta r_n + \Delta r_{n+1})h_{n+1/2}] -$$
$$-D_{n-1/2}\Delta r_{n-1} + (C^+_{n-1/2} - C^-_{n+1/2})\Delta r_n - D_{n+1/2}\Delta r_{n+1} = 0, \quad n = 3, \ldots, N-1;$$
$$B^\pm = Arr' + rp \pm pr'\frac{h}{2}, \quad C^\pm = Ar' + p \pm \frac{2A}{h}r, \quad D = \frac{2A}{h}r, \quad \Delta r_N = 0$$

These equations enable us to determine all the increments Δr_n and the Lagrange multiplier λ and to carry out the next correction of the contour of the fore-body. Quantities of the order of $h\Delta r$ occur in the coefficients of Δr_n in the first equation and in all the coefficients of λ together with the principal terms of order h. Apart from these, system (5) is linear, with a matrix of coefficients, only the first row of which differs from a four-diagonal matrix. In NP with a fixed aspect ratio l with a free volume, the first equation drops out of system (5) while in the remaining ones $\lambda = 0$. The system of N –1 equations thereby obtained is linear with a three-diagonal matrix of the coefficients, which facilitates the construction of the optimum nose shape by the same method [30]. Its volume $\Omega^0 = \Omega^0(l, M_\infty, \kappa)$.

At the beginning of the correction process, the initial contour is chosen in such a way that for fixed l and a volume $\Omega > \Omega^0$ the nose shape satisfies constraints (1). The contours obtained after the next correction satisfy the same constraints. A direct calculation of the flow around any such fore-body gives the wave drag coefficient of the front face c_{x0}, the integral of the pressure forces χ and the flow parameters on the segment t^+f, and consequently the free terms and linear parts of the coefficients of system (5). An iteration procedure for solving it determines all the increments Δr_n. If in the expression for $\Delta\Omega$ (the penultimate relation of (4)) and in the first equation of system (5) we drop the quadratic terms, we obtain a linear system, the solution of which does not require iterations. In this case, however, the need arises to refine the volume of the corrected fore-body.

Each cycle of correction includes a "single-parameter descent", which consist of the following. In addition to the direct calculation of the flow around of the initial fore-body the flow around two corrected fore-bodies is also calculated. The radii of their front faces and the gently sloping parts are replaced by $r_n + s\Delta r_n$ with Δr_n, obtained by the method described, and two values of the parameter s, for example 0.5 and 1. In the s,χ plane a parabola is drawn through the three points corresponding to this s and the initial nose shape ($s = 0$): $\chi(s) = \chi(0) + a_1 s + a_2 s^2$, then the value of $s_m = -a_1/(2a_2)$ which gives a minimum of χ is determined, and from this the radii $r_n + s_m\Delta r_n$ of the new nose shape are found. This nose shape serves as the initial one for the next cycle. With each correction, the increments Δr_n are reduced, while the Lagrange multiplier λ approaches a constant value.

A growth of Ω enivitably results to the situations whet at $n = k_1, \ldots, N$-1 the upper limitation on r from the conditions (2.1) is not satisfied, that is at $r_{ki} < R$ the sum $r_{ki} + \Delta r_{ki} > R$. This means that at the volume chosen, the optimal contur contains a cylindrical segment. In these cases the values Δr_{ki} found have to be replaced with $\Delta r_{ki} = R - r_{ki}$, and the rest values Δr_n have to be found from the equations system (5) excluding the above mentioned equations

The results of building the optimal conturs using the above method are given in [30]. Depending on the aspect ratio and the volume of the fore-body the number of points N in the gently sloping segment of the sought contour varied from 60 to 275. The direct method of optimization showed a high convergence rate: for a specified l and C_Ω when constructing the optimum nose shape it was necessary to calculate the flow around from 25 to 50 configurations.

CONCLUSION

The presented above examples give introducing only about a part of problems solved by small group of the researchers for rather short time. The information on other problems can be received from reviewing of investigations, executed in the same collective [31-36]. A subject of these researches became: optimization of a flight vehicle model with engine [31, 32], optimization of a turbine stage in an approximation of radially equilibrium flow [33], the solving of the important for implementation of inertial thermonuclear fusion problem about energy optimal isentropic compressing from rest to rest [34] and series of papers by the optimization of gasdynamics bearings clearance [35, 36].

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