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ABSOLUTELY OPTIMAL CONFIGURATIONS WITH MAXIMUM LIFT-TO-DRAG RATIO AT HIGH SUPERSONIC FLOW VELOCITY

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Abstract. The problem on the configuration with maximum lift-to-drag ratio in supersonic gas flow is discussed. It is assumed that pressure on the body surface is defined by a local method and friction coefficient is constant.

Corresponding variational problem is stated and solved without isoperimetric conditions and also with given shape and area of the body base section. It is shown that the variational problem extremals are the following: conical surface with very small cone angle (by the order of the friction coefficient), planes perpendicular to the cone surface normal, and planes parallel to Z-axis that form the windward surface of the optimal body. If isoperimetric conditions are not prescribed, absolutely optimal body is two-dimensional wedge. If base area is given then optimal body base shape is a polygon formed by line segments. Optimal parameters are found for bodies with triangular base.

The solution is found analytically and using the numerical technique of local variations.

1. INTRODUCTION

Variational problems on spatial configurations with maximum lift-to-drag ratio (*K*) are discussed in the works[1-6] basing on the Newton's law of resistance with various isoperimetric conditions. With the assumption about small body thickness the analytical solutions were found [1,2]. The Newton formula is a local method for determining the pressure on body surface and it does not take into consideration the influence of Mach number. The method of tangent wedge accounts for this influence, and it provides calculation accuracy comparable with numerical integration of the equation of ideal gas motion[7] and with experimental data[8]. High accuracy and simplicity of the method allow formulating and analytical solving of practically interesting problems on ultimate value of lift-to-drag ratio.

The term "absolutely ideal body" is used by analogy with the work[9], which was a motive of such a study. Optimal body structure and lift-to-drag ratio are discussed below.

2. PROBLEM STATEMENT

Let's consider supersonic flow around arbitrary three-dimensional body and assume that oncoming flow velocity vector $\bar{v}$ is parallel to X-axis of Cartesian coordinate system OXYZ. Y-axis is directed upright, and Z-axis direction completes the right-hand coordinates. The body surface is set as $x + f(y,z) = 0$.

Pressure coefficient on the body surface is defined by the formulas [7]

$$C_p = 2\cos^2(\bar{n},\bar{v})\left\{\frac{\gamma+1}{4} + \left[\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{A^2}\right]^{1/2}\right\}, \quad \cos(\bar{n},\bar{v}) > 0;$$

$$C_p = \frac{2}{\gamma\left(M^2-1\right)}\left[\left(1-\frac{\gamma-1}{2}A\right)^{\frac{2\gamma}{\gamma-1}} - 1\right], \qquad \cos(\bar{n},\bar{v}) \le 0; \qquad (1)$$

$$C_p = -\frac{2}{\gamma(M^2-1)}, \qquad 1-\frac{\gamma-1}{2}A < 0.$$

Here $A = (M^2-1)\cos^2(\bar{n},\bar{v})$, A>0; γ =1,4 – specific heat ratio.
With $A\to\infty$ the dependencies (1) correspond to the Newton formula:

$$C_p = \begin{cases}(\gamma+1)\cos^2(\bar{n},\bar{v}), & \cos(\bar{n},\bar{v}) > 0,\\ 0, & \cos(\bar{n},\bar{v}) \le 0.\end{cases}$$

At low supersonic velocities $A \to 0$, and the dependencies (1) correspond to the formulas of the linear theory of supersonic flow:

$$C_p = \begin{cases}2\cos(\bar{n},\bar{v})\left(M^2-1\right)^{-1/2}, & \cos(\bar{n},\bar{v}) > 0\\ -2\cos(\bar{n},\bar{v})\left(M^2-1\right)^{-1/2}, & \cos(\bar{n},\bar{v}) \le 0.\end{cases}$$

With above assumptions, $\cos(\bar{n},\bar{v})$ is determined by the formula:

$$\cos(\bar{n},\bar{v}) = \left(1+f_y^2+f_z^2\right)^{-1/2} = \left(1+\alpha^2\right)^{-1/2}, \quad \alpha^2 = f_y^2 + f_z^2$$

the dependencies (1) may be written in generalised form:

$$C_p = \Psi(\alpha, M), \qquad (2)$$

where Mach number M is the parameter.

Assume now that the friction coefficient C_f is constant on the body surface.

Aerodynamic coefficients: C_{ya} - lift coefficient and C_{xa} - drag coefficient – are determined by the formulas

$$S\cdot C_{ya} = \iint_\Sigma \left(\Psi(\alpha) - C_f/\alpha\right) u\, dzdy,$$

$$S\cdot C_{xa} = \iint_\Sigma \left(\Psi(\alpha) + C_f\cdot\alpha\right)\, dzdy.$$

Here $S = \iint_\Sigma dzdy$ – base area, $u = \partial f/\partial y$, $w = \partial f/\partial z$, $\alpha^2 = u^2 + w^2$.

Lift-to-drag ratio is defined by the ratio

$$K = C_{ya} / C_{xa} \,. \tag{3}$$

The problem is to find a function *f*(*y*,*z*) that realises maximum of the functional (3). And the base area may be prescribed or not. The functional (3) depends also on Mach number and friction coefficient, which are the problem parameters.

The problem on maximum of the functional (3) is equivalent to the problem on maximum of another functional [10]

$$\begin{gathered} \Phi = \iint\limits_{\Sigma} F(\alpha, u) dz dy, \\ F(\alpha, u) = \left(\Psi(\alpha) - C_f \alpha\right) u - K\left(\Psi(\alpha) + C_f(\alpha)\right) + \lambda. \end{gathered} \tag{4}$$

Here λ – Lagrangian coefficient, α, *u* – functions with variables *y* and *z*.

3. ANALYSIS OF THE VARIATIONAL PROBLEM SOLUTION

The functional (4) depends on the functions *u*, *w* only, so the Euler equations for the extremal surface are written in the next form:

$$\frac{\partial F}{\partial u} = 0, \qquad \frac{\partial F}{\partial w} = 0$$

or

$$\begin{gathered} \Psi(\alpha) - C_f / \alpha + \left[u\left(\frac{\partial \Psi}{\partial \alpha} + C_f / \alpha^2 \right) - K\left(\frac{\partial \Psi}{\partial \alpha} + C_f \right) \right] \frac{u}{\alpha} = 0, \\ \left[u\left(\frac{\partial \Psi}{\partial \alpha} + C_f / \alpha^2 \right) - K\left(\frac{\partial \Psi}{\partial \alpha} + C_f \right) \right] \frac{w}{\alpha} = 0. \end{gathered} \tag{5}$$

Transversability condition takes the form:

$$F = 0\,. \tag{6}$$

The system of equations (5) has two solutions:

first solution

$$\begin{cases} w = 0 \\ \Psi(u) + \left[u \dfrac{\partial \Psi}{\partial u} - K\left(\dfrac{\partial \Psi}{\partial u} + C_f \right) \right] = 0 \end{cases} \tag{7}$$

second solution

$$\begin{cases} u\left(\dfrac{\partial \Psi}{\partial \alpha} + C_f / \alpha^2 \right) - K\left(\dfrac{\partial \Psi}{\partial \alpha} + C_f \right) = 0 \\ \Psi(\alpha) - C_f / \alpha = 0 \end{cases} \tag{8}$$

First solution (7) defines a family of planes parallel to Z-axis, and slope of the planes to XOZ plane is determined by the parameters C_f, M and *K*.

Second solution (8) defines a surface with $\alpha = const$ that is determined by the parameters C_f, M. This surface is a cone with half-angle $\theta = arcctg\alpha$, and its axis coincides with X-axis,

$$\frac{1}{\alpha^2}x^2 - y^2 - z^2 = 0, \tag{9}$$

and a family of planes

$$x + uy + wz + c = 0, \tag{10}$$

with $u^2 + w^2 = \alpha^2 = const$.

Using the Newton formula for determining the pressure coefficient C_p, we get $\alpha \approx k / 4C_f$.

Correspondingly, base shape is a polygon consisting of line segments that may include circular segment with radius l/α, here l – body length. Examples for optimal body configurations are illustrated in Fig.1. Note that cone half-angle $\theta = 1/\alpha \approx 4C_f / k << 1$ is rather small, and the influence of this surface on aerodynamic forces is negligible. In other words, constructing the optimal bodies it is assumed that the cone (9) degenerates into X-axis, and the planes (10) take the form

$$uy + wz + c = 0, \tag{11}$$

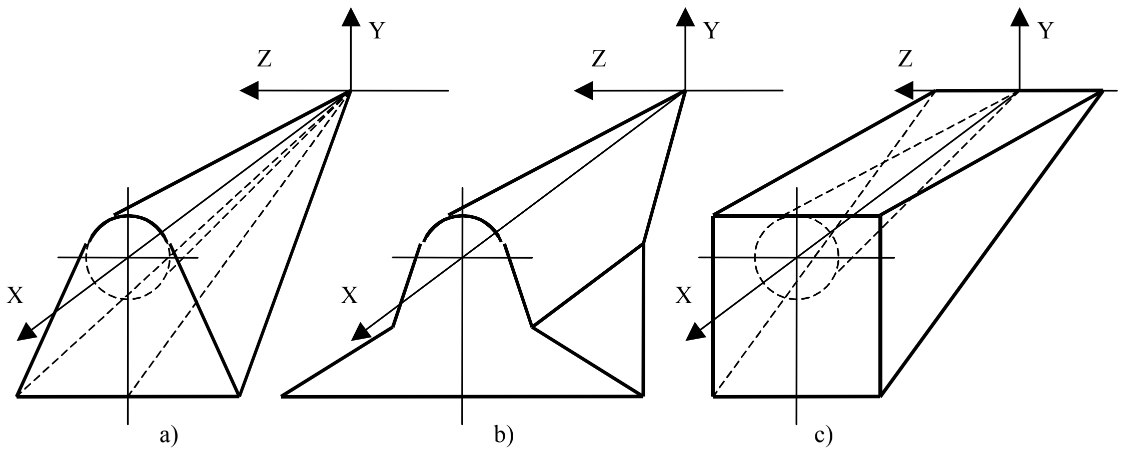

Figure 1. Optimal body configurations

4. GEOMETRICAL PARAMETERS AND LIFT-TO-DRAG RATIO OF ABSOLUTELY OPTIMAL BODIES

Let's consider one of the simplest optimal configurations with the base in form of equilateral triangle. Lift-to-drag ratio of this body is determined by the following formula:

$$K = \frac{\Psi(\alpha) - C_f tg\beta}{\Psi(\alpha)tg\beta + C_f\left(1 + \frac{1}{\sin\varphi}\right)}, \quad (12)$$

β – an angle between the airfoil lower plane and OXZ plane, φ – an angle between the planes YOX and (11); $\varphi = \pi / 2$ corresponds to XOZ plane (Fig.2).

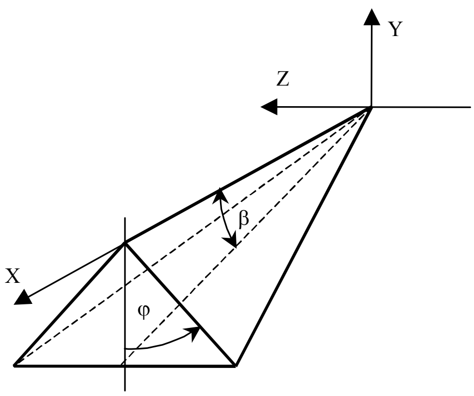

Fig.2. Optimal configuration with equilateral triangle as base section

It is seen that in case of no isoperimetric conditions the function K has boundary point of extremum at $\varphi = \pi / 2$. In other words, the upper surface of absolutely optimal body is XOZ plane itself, and the body is a two-dimensional wedge. The angle β of this wedge depends on type of the function $\Psi(\alpha)$ and assuming that the influence of friction on the coefficient C_{ya} is insignificant then for the Newton formula $\Psi(\alpha) = k\sin^2\beta$:

$$tg^3\beta = 2a(2 + tg^2\beta), \quad a = C_f / k .$$

For thin wedges ($tg^2\beta << 2$), then $tg^3\beta = 4a$, $K = \frac{1}{3}\sqrt[3]{\frac{2}{a}}$.

If base area S is given and body length l=1, then the angles β and φ of absolutely optimal body with triangular base section are determined from the formulas:

$$\left[2a\left(1 + \frac{1}{\sin\varphi}\right)\right]^{2/3} tg\varphi = S, \quad tg^3\beta = 2a\left(1 + \frac{1}{\sin\varphi}\right).$$

If the tangent wedge method was used for determining the pressure coefficient on the body surface then the dependencies of the angle β against φ at different values of Mach number were found by numerical solution of the variational problem on body configuration with maximum lift-to-drag ratio. In this case the algorithm of local variations[11,12] was applied for given triangular shape of base section and angle φ. Corresponding dependencies $\beta(\varphi)$ and $K(\varphi)$ at C_f = 0.0024 and different Mach numbers are illustrated in Fig. 3,4.

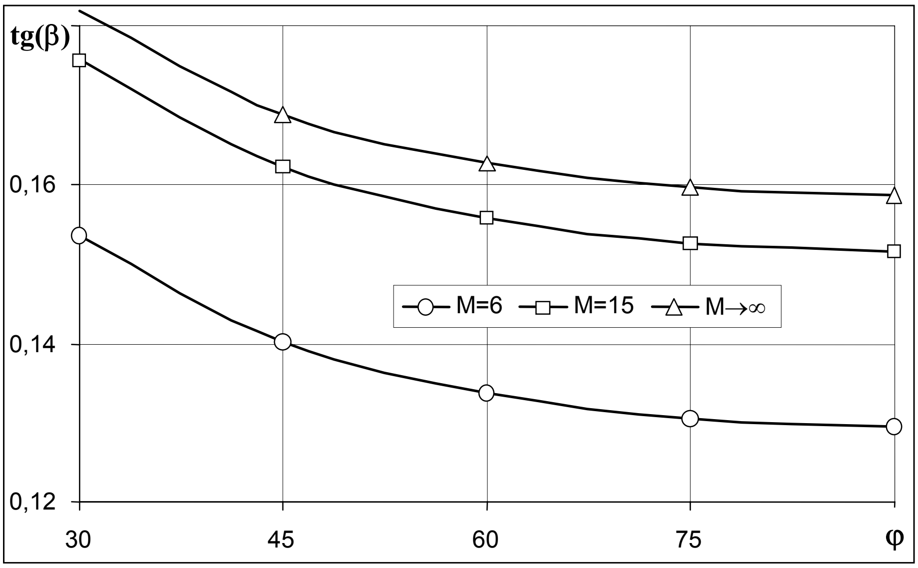

Fig.3.

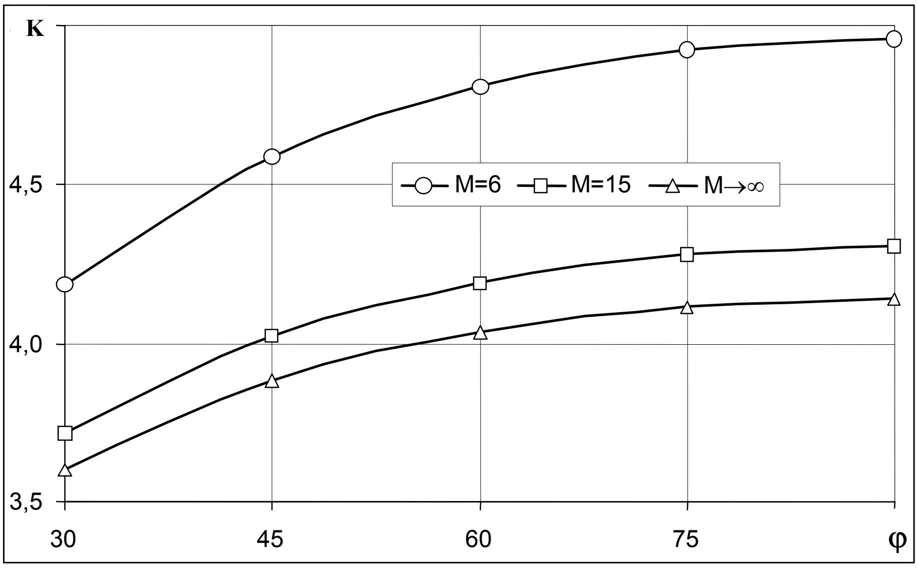

Fig.4. Lift-to-drag ratio of optimal body

5. CONCLUSION

Analysis of optimal body configurations was carried out with the help of the tangent wedge method since it is possible to find the lift-to drag ratio of lifting bodies by this method with high accuracy in the range of high supersonic velocities $M \geq 4$.

Base section of the optimal body is a polygon formed by line segments. The lower (windward) surface of the body is a plane parallel to Z-axis. The upper (leeward) surface is formed by planes parallel to incoming flow velocity vector. If isoperimetric condition is not prescribed then optimal body is two-dimension wedge.

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6. REFERENCES

[1] Maikapar G.I. The wing with Maximum Lift-to-Drag Ratio at Hypersonic Speeds, *PMM*, 1966, v. 30, iss. 1.

[2] Miele A., Hull D.G., Brown S.L. Maximum Lift-to-Drag Ratio of a Slender Flat-Top Hypersonic Body. *Astronautica Acta*, 1967, v.13, №2.

[3] Miele A., Heidemann Y.C., Pritchard R.E. Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds. Part I: Direct Methods. *J.Astr.Sci*., 1968, v.15, №2.

[4] Ho-Yi Huang. Conical Bodies of Given Length and Volume Having Maximum Lift-to-Drag Ratio at Hypersonic Speeds. Part 2: Variational Methods, *J.Astr.Sci*., 1968, v. 15, №3, pp. 118-123.

[5] Perminov V.D. Wings with Optimal Characteristics in Hypersonic Flow. *Izv. AN USSR*, *MJG*, 1969, №6.

[6] Lapygin V.I. Bodies with Maximum Lift-to-Drag Ratio in Hypersonic Flow Proceedings of Institute of Mehanics (Moscow University), 1971, №11, p.p.44-49.

[7] V.I.Lapygin, A.Yu. Galaktionov, M.N. Kazakov, V.A.Mikhalin, D.M. Fofonov Trade-off of aerodynamic configuration for a descent vehicle. Proceeding of the 2nd European conference for aerospace sciences (EUCASS), July 1-7, 2007, Brussels, Belgium.

[8] Singh A., Stollery J.L. Force Measurements on a Delta-Wing Body Combination at Hypersonic Speeds. AIAA-96-4501-CP, 11 p.

[9] Yakunina G.E. In optimal non-conical and asymmetric spatial configurations. *PMM*, 2000, v.64, iss.4., pp.605-614.

[10] A.Miele. The Extremization of Products of Powers of Functionals and Its Application to Aerodynamics. *Astronaut. Acta,* v.12, No1, 1966.

[11] V.V.Kudryavtsev, V.I. Lapygin et al. Hypersonic Aerodynamic Configurations with Maximum Lift-to-Drag Ratio. Fifth European Symposium on Aerothermodynamics for Space Vehicles, 2004, Cologne, Germany. ESA SP-563.

[12] Banichuk N.V., Petrov V.M., Chernousko F.L Local Variations Method for Variational Problems with Non-Additive Functionals. *J.V.M. M.F.* 1969, v.9. №3.