WEST-EAST HIGH SPEED FLOW FIELD CONFERENCE
19-22, November 2007
Moscow, Russia

ACCELERATED METHOD OF AERODYNAMIC SHAPE OPTIMIZATION FOR SUPERSONIC AIRCRAFT DESIGN

Sergey A. Takovitsky*

**Central Aerohydrodynamic Institute (TsAGI), Russia*
Email: c.a.t@tsagi.ru, web page:http://www.tsagi.ru

Key words: optimization, supersonic aircraft, wave drag, sonic boom.

Abstract. The efficient direct optimization method for supersonic aircraft design is presented. The method combines Newton based algorithm with Euler space-marching solver and is capable of handling large number of design variables. Drag and sonic boom signature parameters are used as the objective functions minimized under geometric and aerodynamic constraints. The efficiency of the method is demonstrated on examples of three-dimensional aerodynamic design. Nose shapes that ensure minimum wave drag for specified constraints on the overall size are defined. The simplest shape deformation of a complex plane form wing providing the main part of drag due to lift diminishing is established. The results obtained within the framework of the Euler's equations and simplified flow models are compared. To profile the aircraft wing and fuselage providing optimal sonic boom parameters approaches of theory for sonic boom propagation are included in the method. The initial shock intensity as the primary value of the pressure signature is minimized.

1. INTRODUCTION

Aerodynamic shapes design by means of numerical optimization is fast developing area in aerodynamics of supersonic flying vehicles. On the one hand, the answers on many questions may be obtained by using fairly simple flow models, including models based on the linearized equations of motion. Using these simplified theories, the shape specifics were found and the effects of the constitutive parameters were estimated for the airfoils and bodies of revolution that have minimum wave drag. On the other hand, capabilities of modern computer facilities and methods of mathematical modeling enable to investigate real flying vehicles and to update the results for isolated elements.

To increase efficiency of researches based on sophisticated flow models acceleration of convergence is required. The simplest versatile methods (such as cyclic coordinate wise descent or gradient descent) cannot ensure reliable solution of ill-conditioned optimization problems. Even when relatively few parameters are varied, linear convergence (at the rate of a geometric progression) proves to be insufficient. Since the lines (or surfaces) of constant level of the objective function are extremely stretched, the corrections to parameter values made to reduce the objective function are small, and the relative error of their calculation is too large. The optimization process is terminated at a considerable distance from the optimum point.

The significant acceleration of convergence is reached due to simplification of the variational problem statement[1,2]. A local analysis of the aerodynamic load distribution is used to study the aerodynamic function behavior allowing analytic formulation of the objective function and constraints. On the base of approximations to the true Hessian matrix and gradient vector aerodynamic shape variations that enable the aerodynamic performance to be improved are established. The shape variations are utilized in exact solution. A fast convergence to the optimum in case of the large number of the variables is provided.

2. PROBLEM STATEMENT AND OPTIMIZATION METHOD

Aerodynamic design applications are reduced to constrained minimization of a function of many variables. For an aircraft performing a cruise flight at given Mach number M, altitude H, and weight W (lift $L=W$) the problem may be stated mathematically as:

$$\begin{cases} OF(h) = \min \\ L(h) = const \\ V(h) = const \end{cases} \quad (1)$$

Here OF is the objective function. The geometric constraint limits the internal volume V. The vector of design variables h consists of geometric variables h_i (i=1…N) and angle of attack α. The objective function combines aerodynamic drag and parameters of sonic boom signature.

Firstly the optimization method was developed to minimize aerodynamic drag[1]. Drag was used as the objective function minimized under lift, longitudinal trim and volume constraints. The variational problem is solved in a simplified statement. In the case of small perturbation of supersonic flow the linearized theory allows to connect change of pressure at given surface point with form deflection in its vicinity. This connection can be established both theoretically and through numerical calculation. A summation of aerodynamic loading over all elements of the aircraft surface gives a quadratic approximation of the objective function. On the base of the information on derivatives of first and second order the Newton's method determines shape variations ensuring a quadratic rate of convergence to the optimum. The variations are utilized in exact solution. This approach allowed optimizing aircraft wing subject to many geometric variables (more than 500). The optimization process looked not more than thirty versions of the aerodynamic configuration.

To profile the wing and fuselage providing optimal sonic boom parameters classical approaches are included in the method. Sonic boom pressure signatures for given cruise Mach number, altitude, and aircraft geometry parameters are computed according to geometric acoustic theory with nonlinear effects accounted. Sonic boom propagation in a horizontally stratified atmosphere is modeled. The case of steady flight and no winds is studied. Sonic boom waveforms are found directly below the aircraft flight path.

The problem is solved through two stages. At the first stage, F-function is determined. Lift contribution to sonic boom is taken into account on the base of supersonic area rule theory. One way is to calculate F-function through the integral relation with equivalent area distribution that defined from the aircraft configuration and its lift distribution. It was shown that more convenient equivalent formulation is valid[3]. Equivalent area derivatives are related with disturbances of velocity. Asymptotic dependence of pressure on time (or on longitudinal coordinate) and corresponding F-function could be obtained on the base of flow parameters in a narrow vicinity of the aircraft. So it is not necessary to model flow at long distance from the aircraft flight path. At the second stage, pressure signatures far away from the aircraft at a desired distance from the ground or on the ground are determined using reliable acoustic methods[4,5].

Flow field near the aircraft is modeled within the framework of Euler equations. Conservative form of equations and explicit marching finite-difference method with respect to the longitudinal coordinate are used. In the vicinity of the fuselage nose Euler equations are integrated using a time-dependent procedure. A gas-dynamic properties jump on a head shock wave is allocated strictly. Inside shock waves and other flow discontinuities are treated without tracking their spatial location. The computation mesh is constructed by the multi-zone approach. The maximum number of mesh-points in cross section reaches 50 000. The surface friction drag is determined by a semiempirical calculation method for a turbulent boundary layer.

3. NOSE SHAPES OF MINIMUM WAVE DRAG

One of the classical problems of supersonic aerodynamics is to determine axisymmetric nose shapes which ensure the minimum wave drag for specified constraints on the overall size. The first optimum nose shapes, found by Newton, are characterized by the existence of a front face. A solution of the same problem within the framework of linear theory was obtained by Karman. Aerodynamic shapes, obtained using a simplified formulation of a variational problem, have sections for which the pressure distribution is calculated with a large error. Nevertheless, the efficiency of the approximate laws for the profiling nose shapes has been confirmed by investigations using exact methods of calculation. In the case of power-law bodies ($r=x^m$, where x is measured along the axis of symmetry from the leading point of the body and r is the distance to the axis) calculations using Euler's equations showed that the optimum value of the exponent m lies in the range 0.60 to 0.75[6]. Newton's theory gives value m=3/4 for thin noses.

On the base of more accurate analysis it is shown that the near-optimal bodies have a flat forward face and a power-law generatrix with the exponent equal to 2/3. The only parameter dependent on the free stream Mach number and the nose aspect ratio $\lambda=L/2R$ (L – the nose length, R – the base radius) is the forward face radius r_1. For the nose generatrix the following dependence of the radius on the longitudinal coordinate is proposed:

$$r = \frac{L}{2\lambda}\left[\left(\frac{2\lambda r_1}{L}\right)^{3/2} + \left(1-\left(\frac{2\lambda r_1}{L}\right)^{3/2}\right)\frac{x}{L}\right]^{2/3} \tag{2}$$

By varying r_1 one can solve the problem of the extremum for a function of a single variable and thus find the minimum drag nose. The results for bodies with aspect ratios λ=1, 2, 4, 6, and 8 at M=2, and 4 are presented. In calculating the drag coefficient C_D the forces were divided by the freestream velocity pressure and the base area. For the sake of comparison, the data for minimum-drag noses constructed numerically using the direct optimization are shown also[7,8].

As compared with the truncated power-law bodies, the optimal bodies give a gain in drag not greater than 3% for M=2 and not greater than 1% for M=4 (figure 1,a). The greatest discrepancy is observable at low (λ=1) and high (λ=8) aspect ratios. The ratio of the forward face radius to that of the base decreases with increase in the aspect ratio. The difference in this geometric parameter is not more than 15% (figure 1,b).

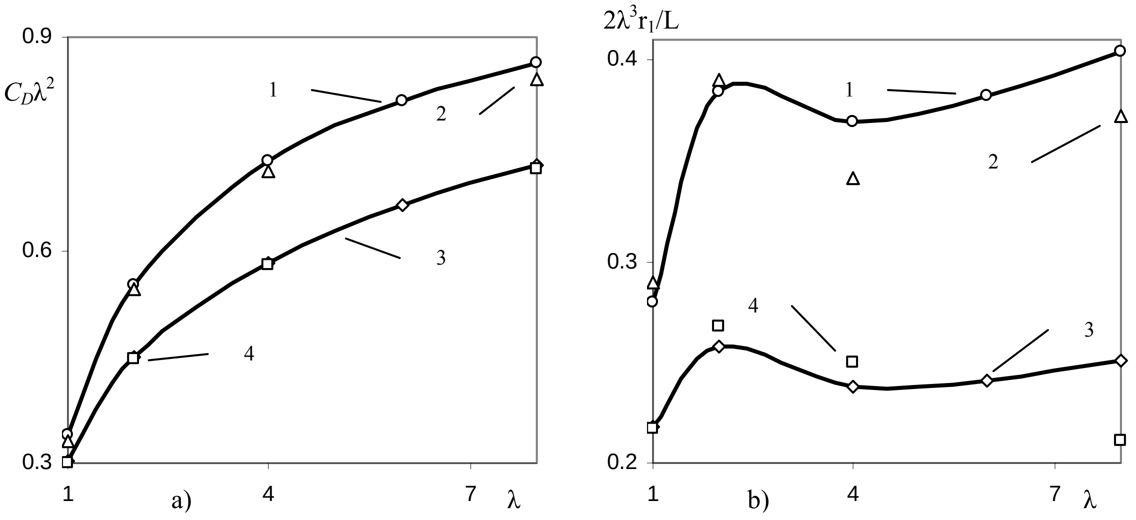

Figure 1. a) Drag coefficient C_D; b) Forward face radius r_1.
1 – power-law bodies (M=2); 2 – optimum bodies (M=2);
3 – power-law bodies (M=4); 4 – optimum bodies (M=4).

Optimal shapes of noses of small aspect ratio differ significantly from shapes defined on the base of Newton pressure equation. On values of drag relative difference exceeds 25%. The radius of the front face of Newton body is twice smaller. With increase of Mach number and of aspect ratio the reliability of Newton theory grows. At λ=4 difference on drag is smaller than 6%. But geometrical parameters are still different significantly.

4. COMPLEX PLANE FORM WING

Optimization of the median surface of the complex plane form wing (proposed for aircraft Tu-144) is performed. Both spatial changes to the wing shape and simple, near to conical, deformations are investigated.

The wing projection on the base plane is broken into triangular elements. For the wing half 26 longitudinal sections are allocated, in each of which the median line of the airfoil is formed by 20 segments. The trailing edge was not deformed. Number of geometrical parameters h_i (displacement of central points on the normal to the base plane) is N=519. The plane form does not change.

The opportunity of reduction of lift induced drag by means of the simplest deformations established earlier for delta wing is investigated additionally. The wing surface is represented by four flat elements joined along lines passing trough the wing top. The elements have identical angles at the top. In this case deformation is defined by two geometrical parameters. The condition of plane form conservation is replaced by the condition of wing surface conservation.

Optimization is performed at Mach number M=2.1 and lift coefficient C_L=0.1. Thus the wing has subsonic leading edges. The received results are presented in comparison with data of research in linear statement[9].

Geometrical characteristics are twist angle of airfoils and median lines of airfoils. On figure 2 comparison of twist angle distribution on wing span is given. Twist angles differ on absolute value and have different laws of change on span. Dependence $\varphi(s)$ for wing defined in nonlinear statement is close to linear. The wing made of four flat elements is characterized by a constant twist angle of console part. At the same time, wings have similar integral aerodynamic characteristics. It confirms ill-conditionality of optimization problems. There are wing form variations which weakly influence on aerodynamic characteristics of the wing.

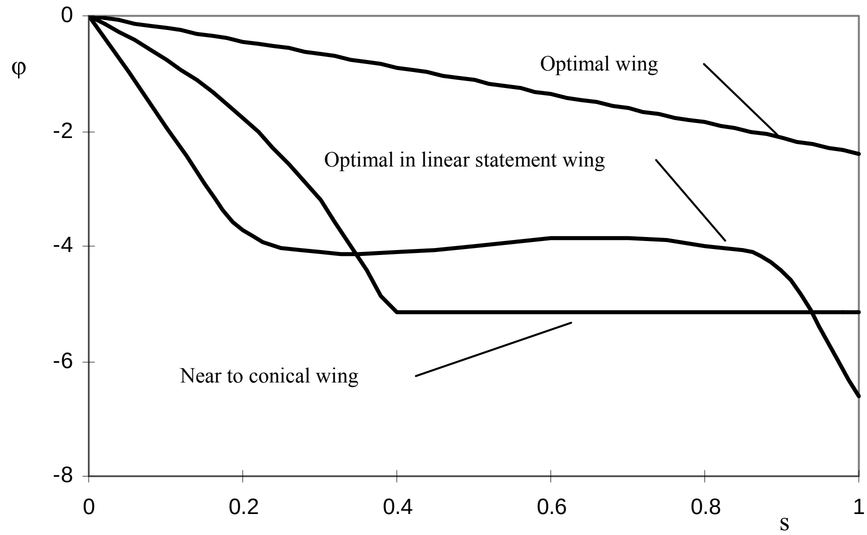

Figure 2. Twist angle dependence on wing span

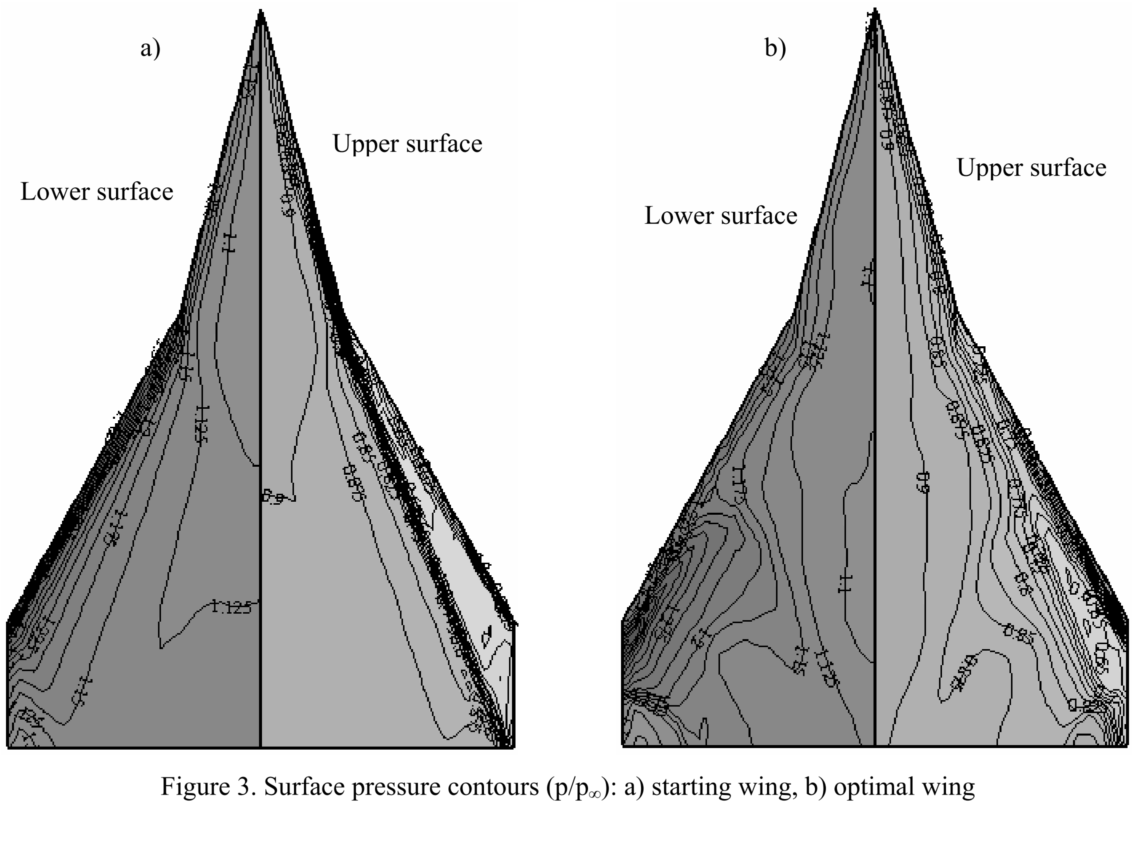

Figure 3. Surface pressure contours (p/p_∞): a) starting wing, b) optimal wing

The median lines of airfoils of optimum wings have small relative sizes of the maximal concavity. The central airfoil of the optimum in linear statement wing has characteristic *S*-figurativeness. At the wing constructed in numerical calculation this feature is expressed less sharply. The console part of the wing is convex in the lee side.

Optimum deformation of the wing leads to more uniform distribution of loading on wing span. Pressure difference in a vicinity of the leading edge decreases (figure 3). Flat and optimum wings have similar lift and moment characteristics. A derivative of lift coefficient on attack angle and aerodynamic centre position change slightly. Comparison of induced-drag polar shows the superiority of wings with a nonplanar median surface. Relative reduction of aerodynamic drag achieves 22%. At the same time near to conic deformation of the wing allows diminishing drag due to lift on 20%. It confirms importance of researches on definition of the simple deformations in problems of aerodynamic forms optimization.

5. SUPERSONIC CIVIL AIRCRAFT

Numerical investigation of the aircraft designed for cruise flight at Mach number M=1.8, altitude H=16 kilometers is performed. Aircraft weight is W=50 000 kilograms. The fuselage length is L=40 meters. Plane view wing area is S=160 square meters. The fuselage interior volume is V=120 cubic meters.

Flight conditions affected on wing plan form that performed with cranked leading-edge and trailing-edge (figure 4). The wing is assumed to have the constant shape in the plane view. During optimization the fuselage of round cross section was modified by means of internal volume redistribution in longitudinal direction, and fuselage axis of symmetry cambering.

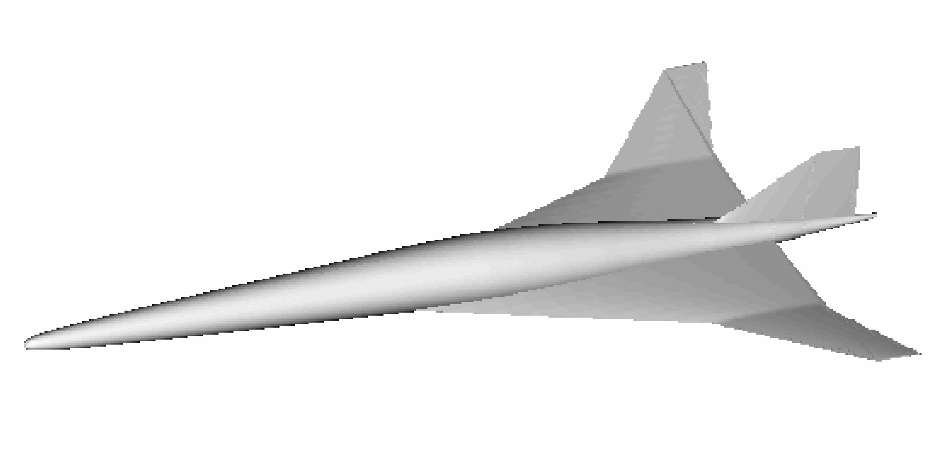

Figure 4. Supersonic aircraft

The wing is represented by a set of elements making its surface up. Moving these elements one can model the diversity of wing geometry. The partition into elements is introduced to wing projection onto the base plane. The base plane is defined as plane, intersecting the vertical symmetry plane at right angle along the inboard chord of the wing. For the wing designed 21 longitudinal sections are allocated, each of which is partitioned into 20 segments. The nodal points are not condensed to leading and trailing edges and define the apexes of triangular elements forming the upper and lower wing surfaces. Geometrical parameters h_i (i=1…N) are stated as displacements of nodal points in the normal direction of the base plane. As a result, surface of each half wing is partitioned into 1600 elements. The number of parameters is equal N=821. The trailing edge consists of three line segment. Displacement of the trailing edge is constrained. The wing edges are assumed to be sharp. The flat wing with symmetrical profiles is taken as a starting one. The relative thickness of the wing profiles is 3%. During optimization it is allowed to redistribute interior volume in longitudinal direction only.

The optimal wing forms were found for different problem statements. At the first statement aerodynamic drag was used as the objective function (wing 1). At the second statement the objective function was chosen as displacement between F-function (or derivatives of equivalent cross section area) distributions for the aircraft and the optimal equivalent body of revolution (wing 2). At all cases the aircraft has equal fuselage corresponding to the second problem statement.

The results are shown as comparison of geometrical, aerodynamic and sonic boom dependences for the starting variant of the wing and optimum variants received by optimization for different statements of the problem. Plane view wing area and fuselage length are adopted as reference values.

The effects of aircraft wake and engine exhaust are neglected. Solutions were found without restriction on rear shock. Fore part of pressure signature with initial shock was analyzed. The initial shock intensity as the primary value of the pressure signature is minimized. In this case overpressure levels are allowed to rise following the initial shock. The rate of rise in this signature is controlled and equals the rate of decrease after maximal overpressure in absolute value. Sonic boom minimizing equivalent area distribution is determined by three parameter power law[10].

Results of sonic boom modeling (F-function and pressure disturbance on the ground) for the starting aircraft and the equivalent body of revolution are compared. The starting aircraft has the fuselage constructed in accordance with Sears-Haack body. Theoretical analysis shows opportunity of significant weakening of the initial shock. The initial shock intensity is 67 Pa for the starting aircraft and 25 Pa for the optimum equivalent body of revolution. The ground reflection factor equals 2.

Geometric parameters of the wings are given in figures 5 and 6.

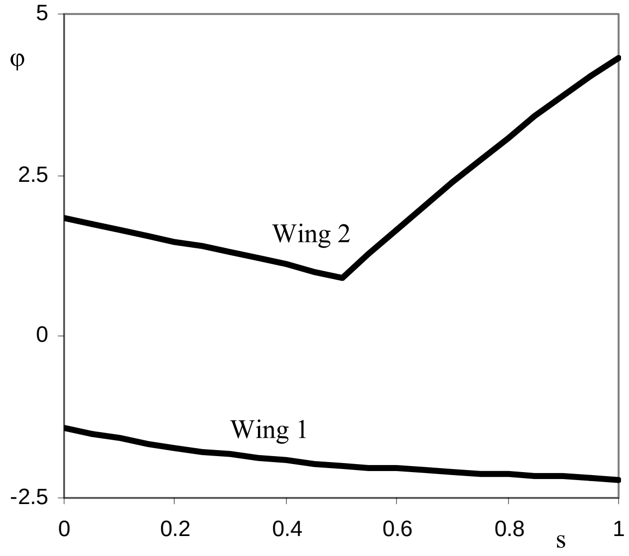

Figure 5. Twist angle

Distributions of twist angle φ over wing span *s* are rather different for wing 1 and wing 2 (figure 5). Wing 1 is characterized by negative values of twist angle. In absolute value φ increases from root chord to tip chord. Twist angle of wing 2 decreases along span of inboard part. After leading edge kink twist angle increases. Near the tip edge sections have maximal twist.

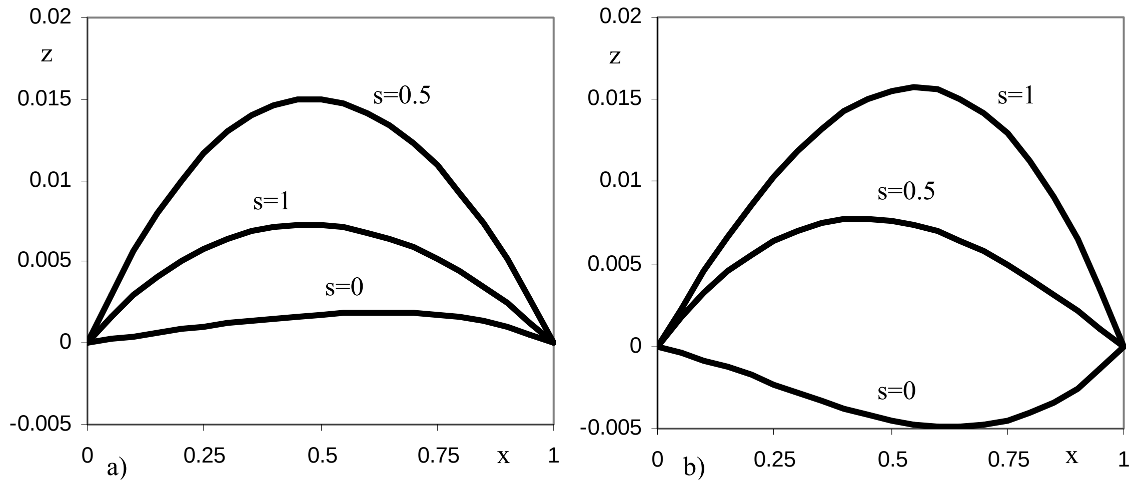

Figure 6. Median lines: a) Wing 1; b) Wing 2

Figure 6 represents median lines of the wings for three longitudinal sections. Span of the half wing is adopted as reference value. Root section of wing 2 has windward camber. Displacement of the trailing edge provides positive V dihedral of wing 2. Inboard part of wing 1 is declined downward.

Figure 7 compares the aerodynamic characteristics. The wing optimization at the problem statement 1 allows increasing of the lift-to-drag ratio L/D of the aircraft at 12%. Cruise flight lift coefficient is C_L=0.13.

Results of sonic boom modeling for the aircrafts with optimal wings and for the optimal equivalent body of revolution are represented on figure 8. Deformation of wing (problem statement 2) aligns distributions of F-function. As result pressure disturbance signature on the ground consists of a number of successive weak shocks. Intensity of initial shock is 28 Pa. In case of wing 1 pressure difference increase to 53 Pa.

6. CONCLUSIONS

Using of square-law approximation of the objective function at the stage of the variational problem solving provides high speed of convergence. The developed optimization method allows designing at a large number of variables, with different geometrical and aerodynamic constraints. The efficiency of the method is demonstrated on examples of fuselage and wing profiling. For the supersonic civil aircraft opportunity of lift to drag increase and sonic boom weakening are investigated.

7. ACKNOWLEDGEMENTS

This work was supported by the Russian Foundation for Basic Research, project No. 05-01-00691.

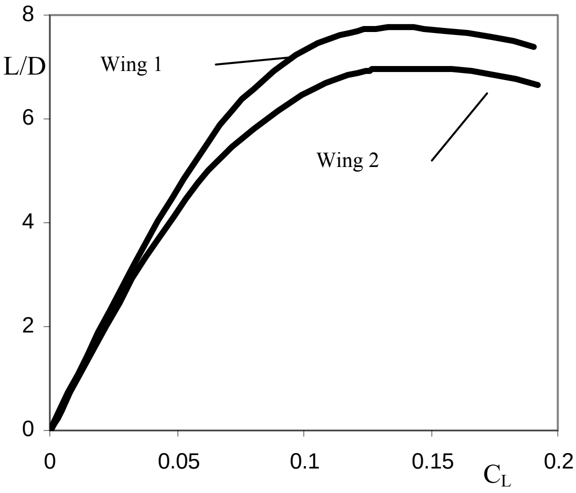

Figure 7. Lift to drag ratio

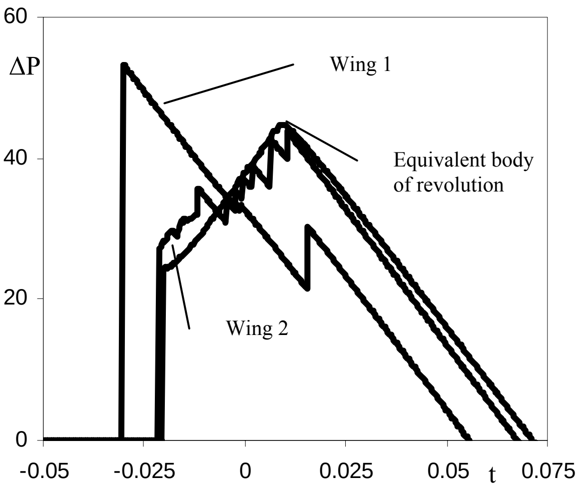

Figure 8. Pressure signature

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