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ABOUT BOUNDS OF APPLICABILITY OF CONTINUUM APPROACH TO THE PROBLEMS OF HYPERSONIC RAREFIED FLOW OVER BODIES

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Abstract. Two hypersonic rarefied flow problems are considered: the flow near a flat plate and the flow over blunted bodies. The first problem – the rarefied flow near a flat plate in a presence of heat transfer – is solved by the direct simulation Monte Carlo method and by numerical calculation of the Krook kinetic equation. These solutions are compared with the analytical solution of boundary layer equations with wall conditions taking into account slip effect. The second problem – the hypersonic rarefied flow over blunted bodies was studied by numerical solution of full and thin viscous shock layer equations with and without taking into account slip velocity and temperature jump at a wall and by asymptotic solution of thin viscous shock layer equations. Applicability of continuum-flow solutions is investigated by comparison with results obtained by the direct simulation Monte Carlo methods for freestream conditions corresponded to Space Shuttle reentry trajectory at altitudes 75-150 km.

1. INTRODUCTION

Hypersonic rarefied flow problems, typical for space vehicles reentry at high altitudes, are investigated: the flow near a flat plate and the flow over blunted bodies. In transitional to free-molecule flow regime continuum-flow models, such as full and parabolic Navier-Stokes equations and viscous shock layer, give heat-transfer and skin-friction coefficients that increase unlimitedly as Reynolds number decrease. Taking into account slip wall conditions reduce these coefficients and thus extend a range of applicability of continuum-flow models to lower Reynolds numbers, but does not remove their tendency to increase. So in a transitional flow regime the direct simulation Monte Carlo method (DSMC) is widely applied or kinetic approach is used. Recently various hybrid methods have being developed: these methods join the solution of kinetic equations - Boltzmann equation or its simplified models - or the solution, obtained by DSMC method, with the solution of continuum equations - Navier-Stokes equations or their simplified models; at that there is a difficulty, connected with coupling of different solutions.

Restriction on application of continuum-flow models in a rarefied flow does not exclude using of continuum approach for prediction some flow parameters. In present work it is shown that numerical and asymptotic solutions of thin viscous shock layer equations at low Reynolds number for pressure, heat-transfer and skin-friction coefficients on a blunted body are in a good agreement with solutions obtained by the DSMC method in a transitional flow regime and they give correct free-molecule limits for these coefficients as Reynolds number decreases to zero. It is shown also on basis of comparison with DSMC results that correct taking into account slip velocity and temperature jump in wall boundary conditions extends the range of applicability of full viscous shock layer model up to altitudes ~ 120 km (at nose radius $\approx$ 1 m) for freestream conditions corresponded to Space Shuttle reentry trajectory. The solutions of the problem of the hypersonic rarefied flow near a sharp flat plate, obtained by the DSMC method and by numerical calculation of the Krook kinetic equation, are compared with the solution obtained by using the continuum approach within the framework of the boundary layer equations with wall conditions taking into account slip effects, and it is shown, that boundary layer solution qualitatively correctly and quantitatively satisfactorily predict skin friction coefficient in a transitional flow regime.

2. HYPERSONIC RAREFIED FLOW NEAR A FLAT PLATE

2.1. Solution by the direct simulation Monte Carlo method

The problem of hypersonic rarefied flow near a sharp flat plate was solved by the direct simulation Monte Carlo method[1]. A length of a plate $L = 0.9\ m$, freestream temperature $T_\infty = 300^\circ K$, wall temperature $T_w = 500^\circ K$, freestream number concentration $n_\infty = 10^{22}\ m^{-3}$. Calculation area was divided into cells: 100 on axis x, directed along a freestream velocity vector, and 60 on axis y normal to a flat plate. Mixed specular – diffusion reflection conditions are accepted on the surface of a plate. 5000 time steps are needed to receive a steady solution for all macroscopic parameters. Velocity, density, temperature distributions, skin-friction and heat-transfer coefficients were obtained for freestream Mach number $M = 4.1$ and 10; at that freestream Knudsen number $Kn_\infty = 0.0143$ and 0.0118 and Reynolds number $Re_\infty = 434$ and 526.

Surface pressure, heat-transfer and skin-friction coefficients (c_p, c_h, c_f) in dependence on x (distance from the front edge of calculation area) for $M = 10$ are shown in Figure 1.

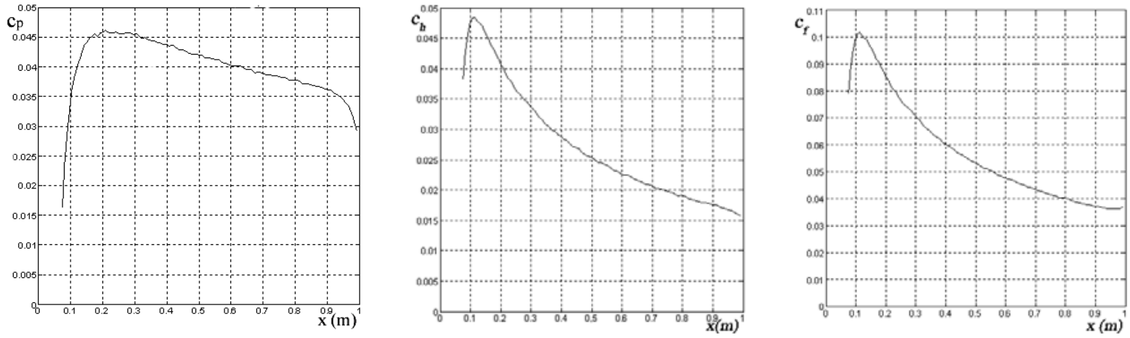

Figure 1. Pressure, heat-transfer and skin-friction coefficients along a flat plate

In Figure 2 the profiles of tangential velocity (U/U_∞) at different x′ (distance from the front edge of a plate) are presented in dependence on self-similar variable $y' = y\cdot(U_\infty/(\nu x'))^{1/2}$ for $M = 4.1$ and $M = 10$, ν - kinematic viscosity coefficient . The self-similarity can be observed at some distance from the front edge of a plate.

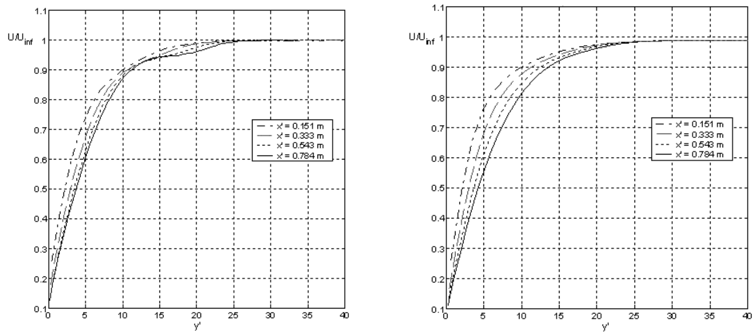

Figure 2. Velocity profiles for $M = 4.1$ (left) and $M = 10$ (right)

2.2. Comparison with the boundary layer solution

The solution of the problem of the hypersonic rarefied flow near a flat plate, obtained by the DSMC method, was compared with the solutions obtained by using the continuum approach within the framework of the incompressible boundary layer: with wall conditions taking into account slip effects[2] – Hasimoto solution and without taking into account slip effect – Blasius solution. For $M = 4.1$ the total skin-friction coefficient for a flat plate with a length equal 0.9 m, predicted by the DSMC method, is $C_D = 0.014$, and Blasius solution gives $C_D = 0.016$. For $M = 10$ the total skin-friction coefficient for a flat plate, predicted by the DSMC method, is $C_D = 6.9\cdot10^{-4}$, and Blasius solution gives $C_D = 6.5\cdot10^{-4}$. So there is a good agreement between DSMC and continuum results for total skin-friction coefficient.

The distribution of a local skin-friction coefficient along the plate is shown in Figure 3 for $M = 4.1$ and $M = 10$. Three solutions are compared: DSMC solution, Blasius solution and Hasimoto solution[2]. At $M = 4.1$ Blasius solution is in a good agreement with DSMC solution except for neighbourhood of the front edge of a plate, where Blasius solution is incorrect, unlimitedly increasing as $x' \to 0$. As distinct from Blasius solution, Hasimoto solution give finite quantity for skin-friction coefficient at front edge of a plate. At M = 10 quantitative agreement of the solutions is not so good apparently because of the influence of compressibility, but qualitative behaviour is similar: the solutions decrease as $1/\sqrt{x'}$.

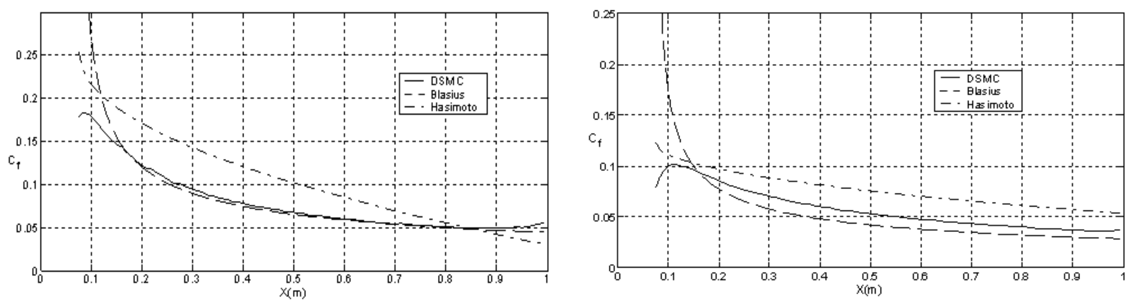

Figure 3. Skin-friction coefficient along a flat plate for $M = 4.1$ (left) and $M = 10$ (right)

2.3. Solution of the Krook kinetic equation

The numerical calculation of Krook[3] kinetic equation was also carried out to solve the problem of hypersonic rarefied flow near a flat plate. Because of its simplicity, the kinetic model BGK[3], known as Krook equation, is widely applied. Unsteady Krook equation was solved by using implicit numerical Euler scheme of the first order approximation on space and time coordinates. The model of hard spheres was used for molecule interaction. Diffusion molecule scattering with full thermal accommodation to surface temperature was accepted as condition on the surface of a plate. Freestream conditions were defined by number density n_∞, temperature T_∞ and velocity U_∞; surface temperature was equal freestream temperature. Calculation carried out at freestream Knudsen number $Kn_\infty = 0.01, 0.1, 1, 10$ and velocity ratio $S_\infty = \sqrt{(5/6)}M_\infty = 5, 10, 15, 20$.

As Mach number M_∞ increase, a temperature undergoes strongly in disturbed flow near a plate: at $S_\infty = 5$ and $Kn_\infty = 1$ maximum of a temperature behind a shock is about 4.6, while at $S_\infty = 20$ it is about 68. At that a density changes not so strongly (from 1.5 to 6).

A transition from low *Kn* number to large is of special interest. Knudsen layer thickness is of order of free path. In the continuum flow regime (Kn_∞ = 0.01) Knudsen layer thickness is less than a cell of computational grid. The values of macroscopic parameters nearby a plate correspond to tens of free paths. As *Kn* number increase, a cell of computational grid becomes comparable with Knudsen layer thickness. It enables to obtain more detailed information about macroscopic parameters close to a plate surface. Some distributions of various macroscopic parameters – temperature T/T_∞, density ρ/ρ_∞ , velocity U/U_∞ - along a plate (x axes) and along a normal to a plate (*y* axes) at various *Kn* numbers are shown in Figures 3, 4 for $S_\infty = 5$; $x = 2$ corresponds to a front edge of a plate (*x* and *y* are scaled by a length of a plate *L*).

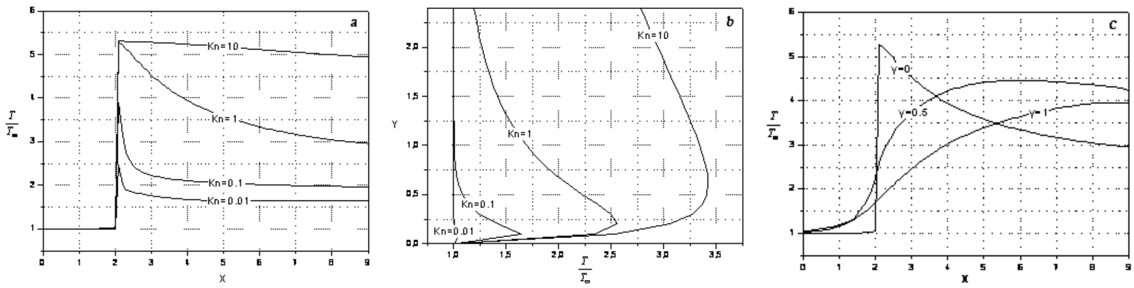

Figure 3. Temperature versus *x* at $y = 0$ (*a*) and versus *y* at $x = 2$ (*b*) at various *Kn* numbers and versus *x* at various *y* at $Kn = 1$ (*c*)

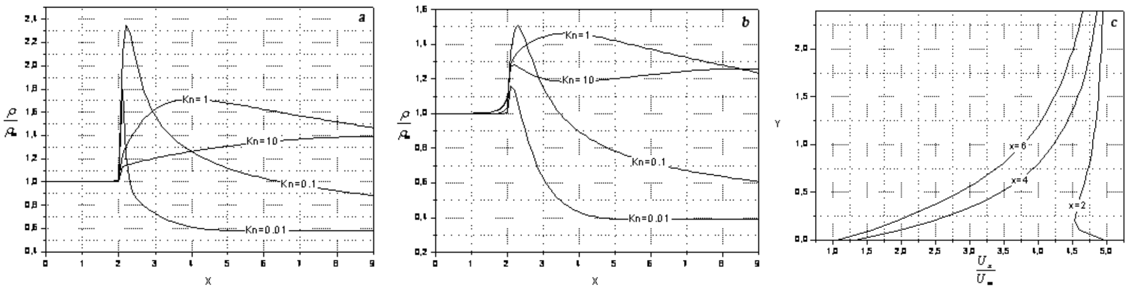

Figure 4. Density versus *x* at $y = 0$ (*a*) and $y = 0.1$ (*b*) at various *Kn* numbers and velocity versus *y* at various *x* at $Kn = 1$ (*c*)

The changing of temperature jump and slip velocity in dependence on *Kn* number have been analysed, so Figure 3a demonstrates temperature jump dependence on longitudinal coordinate x at various *Kn* numbers. Temperature jump increases as *Kn* number increases, it agrees with a fact that temperature jump is proportional to a free path. At $Kn_\infty = 0.01$ and $Kn_\infty = 0.1$ a flow is closed to continuum, and interaction of molecules with a surface is enough strong, so a value of jump changes notably along a plate. In a transitional regime at $Kn_\infty = 1$ interaction is slackened, and a value of jump is monotonically decreased along a plate. Figure 4c shows a large slip velocity nearby the front edge of a plate. At $Kn_\infty = 10$ a flow is closed to free molecular, at that interaction of molecules with a surface is determined by single collisions and a value of jump is nearly does not change along a plate. At $Kn_\infty = 0.01$ and $Kn_\infty = 0.1$ there are a shock and rarefaction area downstream, at $Kn_\infty = 1$ and $Kn_\infty = 10$ a shock is disappeared because of flow rarefaction (Figures 4a and 4b).

3. THE HYPERSONIC RAREFIED FLOW OVER BLUNTED BODIES

The second problem – the hypersonic rarefied flow over blunted bodies was studied within the framework of full viscous shock layer (VSL) and thin viscous shock layer (TVSL) with wall boundary conditions with and without taking into account slip velocity and temperature jump by numerical and asymptotic methods. TVSL and VSL equations, proposed at first[3,4] for high *Re* number, later on were derived from Navier-Stokes equations also at low *Re* number on an assumption of small parameter χ[5,6]. At that asymptotically correct TVSL model imply wall boundary conditions without slip effects.

Numerical solutions of thin and full viscous shock layer equations were obtained by using the low-iterative high-resolution fully coupled implicit space-marching procedure; the accelerated method of global iterations on an elliptical component of pressure gradient was elaborated to take into account upstream influence. New splitting of a tangential pressure gradient into hyperbolic and elliptic components was employed. At a shock modified Rankine-Hugoniot relations were used. Different wall boundary conditions with taking into account slip velocity and temperature jump were considered. After comparing results of calculations with various wall conditions with DSMC results the correction factor k_T for temperature jump in usual wall conditions[8] was founded, so the following boundary conditions have been chosen at a wall

$$u = \frac{2-\theta}{\theta}\sqrt{\gamma\frac{\pi}{2}}\frac{\mu}{\rho\sqrt{(\gamma-1)h}}\frac{\partial u}{\partial y},$$
$$T = T_w + k_T\frac{2-\alpha'}{\alpha'}\frac{2\gamma}{\sigma(\gamma+1)}\sqrt{\gamma\frac{\pi}{2}}\frac{\mu}{\rho\sqrt{(\gamma-1)h}}\frac{\partial T}{\partial y}, \quad k_T = \frac{1}{1.57}. \tag{1}$$

Here u – velocity, h – enthalpy, T – temperature, T_w – wall temperature, $\mu \sim T^\omega$ - viscosity coefficient, γ - specific heat ratio, σ – Prandtl number, θ and α' - diffusion reflection coefficient and accommodation coefficient - were accepted equal a unit.

The asymptotic solution of TVSL equations at low *Re* number is obtained for axisymmetric ($\nu = 1$) and plane ($\nu = 0$) flows for three regimes I, II, III. The solution for heat transfer coefficient c_H, skin-friction coefficient c_f and pressure at a wall p_w in dependence on flow parameters Re, ε, σ, ω, T_w and geometric parameters α, r_w, R, ν is

$$c_H = \sin\alpha\left[1-\left(\frac{1+\nu}{\beta+\nu}\phi-\frac{1}{3}\right)\sigma\tau\right]+O(\tau^2),\quad \beta=\frac{r_w}{R\cos\alpha},\quad \beta^*=\frac{2r_w}{(1+\nu)\sin\alpha\cos\alpha},$$

$$c_f = 2\sin\alpha\cos\alpha\left[1-\left(\frac{1+\nu}{\beta+\nu}\phi+\frac{2\beta}{3(\beta+\nu)}-\frac{1}{3}\right)\tau\right]+O(\tau^2),$$

$$p_w = \sin^2\alpha-\frac{2r_w\cos\alpha}{3(1+\nu)R}\tau+O(\tau^2),$$

$$\text{Regime I:}\quad \tau=(\sigma^{1-\omega}\varepsilon\,\mathrm{Re}\,\beta^*)^{1/(1+\omega)},\quad \phi=\frac{1}{2-\omega},$$

$$\text{Regime II:}\quad \begin{array}{l}\tau=(\sigma^{1-\omega}\varepsilon\,\mathrm{Re}\,\beta^*)^{1/(1+\omega)}(1+\lambda)^{(1-\omega)/(1+\omega)},\quad \phi=\dfrac{(1+\lambda)^{2-\omega}-\lambda^{2-\omega}}{(2-\omega)(1+\lambda)^{1-\omega}},\\ T_w=(\sigma^2\varepsilon\,\mathrm{Re}\,\beta^*)^{1/(1+\omega)}\lambda(1+\lambda)^{(1-\omega)/(1+\omega)}\end{array} \qquad (2)$$

$$\text{Regime III:}\quad \tau=(\varepsilon\,\mathrm{Re}\,T_w^{1-\omega}\beta^*)^{1/2},\quad \phi=1,$$

$$\lim_{\mathrm{Re}\varepsilon\to 0}c_H=\sin\alpha\qquad \lim_{\mathrm{Re}\varepsilon\to 0}c_f=2\sin\alpha\cos\alpha\qquad \lim_{\mathrm{Re}\varepsilon\to 0}p_w=\sin^2\alpha,$$

I: $Re\varepsilon >> T_w^{(1+\omega)}/\beta^*$ II: $Re\varepsilon = O(T_w^{(1+\omega)}/\beta^*)$ III: $Re\varepsilon << T_w^{(1+\omega)}/\beta^*$.

Here $Re = \rho_\infty V_\infty R_0/\mu(T_0)$, ρ_∞, V_∞ and T_0 – freestream density, velocity and stagnation temperature, $\varepsilon = (\gamma-1)/2\gamma$, $T_w T_0$ – wall temperature, $\rho_\infty V_\infty^2 p_w$ - pressure at a wall, R_0 – nose radius, RR_0 – radius of curvature, α – an angle between a tangent to a surface contour and freestream velocity $\mathbf{V}_\infty$, $r_w R_0$ – a distance from a surface to an axis of symmetry.

Regime I corresponds to a strongly cold wall and solution in this case does not depend on T_w; regime I can be considered as limit case of regime II at $\lambda \to 0$. At $\omega = 1$ solutions for all three regimes coincide with each other. As Re, or $\tau \to 0$ pressure, heat-transfer and skin-friction coefficients approach their correct free-molecule limits at a unit accommodation coefficient.

Asymptotic and numerical thin and full viscous shock layer solutions with and without slip boundary conditions were compared with results obtained by the direct simulation Monte Carlo methods[9, 10] and with free-molecule flow solution. Freestream conditions corresponded to Space Shuttle reentry trajectory at altitudes 75-150 km, $V_\infty = 7.5$ km/s. Some results of comparison are represented in Figure 5 for heat-transfer coefficient distributions along a sphere with $R_0 = 0.0254$ m, $T_w = 0.07$. Numerical solutions of VSL and TVSL equations and asymptotic solution are obtained at $\gamma = 1.4$, $\omega = 0.5$, $\sigma = 0.71$.

Altitudes 90, 100 and 110 km in Figure 1 correspond to freestream Kn_∞ numbers 0.906, 8 and 30.35 and to Re numbers 4.26, 0.665 and 0.094. Comparisons shows that taking into account wall slip effects (conditions (1)) extends the range of applicability of full viscous shock layer model up to $Kn_\infty = 8$ ($Re = 0.665$, altitude 100 km at $R_0 = 0.0254$ m) in a stagnation region. At 110 km VSL give incorrect results, and TVSL give results coinciding with DSMC solution and very closed to free-molecule solution. Comparison with DSMC results[10] shows, that at $R_0 = 1.36\ m$ VSL model with slip effect give good heat-transfer prediction up to 120 km. It is shown also that thin viscous shock layer solutions – numerical and analytical – for skin friction and heat transfer coefficients are in good agreement with DSMC[9, 10] results in transitional flow regime, at altitudes higher than 100 km at $R_0 = 1.36\ m$ and higher than 80 km at $R_0 = 0.0254\ m$ and the values of these coefficients approach correct free-molecule limits (at a unit accommodation coefficient) as $Re \to 0$. Results obtained give grounds for creating the hybrid method for heat transfer and skin friction prediction in hypersonic flow over blunted bodies at any Reynolds numbers, which would be based on using only continuum-flow models.

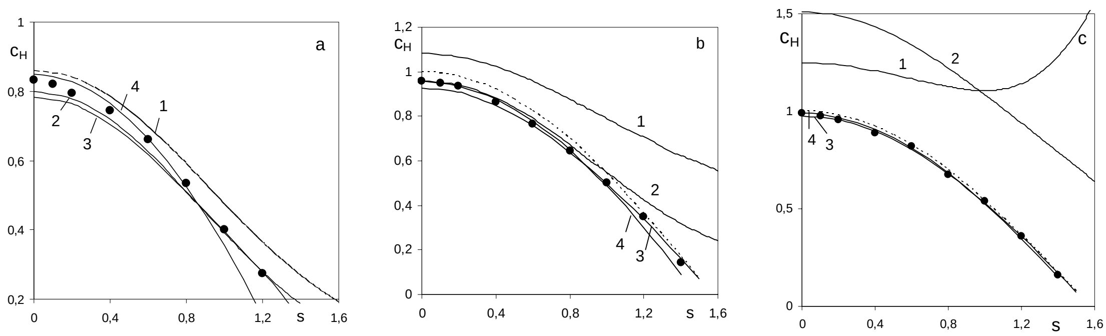

Figure 5. c_H distributions along a sphere (s – a length of arc from a stagnation point) at altitudes 90 (a), 100 (b) and 110 (c) km. Lines 1 and 2 – VSL solutions without and with slip effect; 3 and 4 – numerical and asymptotic TVSL solutions; dotted line – free-molecule flow solution; black dots – DSMC[9] solution

4. CONCLUSION

The problem of the hypersonic rarefied flow near a flat plate has been solved by the direct simulation Monte Carlo method and by numerical calculation of the Krook kinetic equation. Boundary layer solution with wall conditions taking into account slip effects qualitatively correctly predict skin friction coefficient on a plate surface. For hypersonic rarefied flow over blunt bodies the continuum model of thin viscous shock layer give good prediction for heat transfer and skin friction coefficients in the transitional flow regime. The model of full viscous shock layer with slip boundary conditions give good heat transfer prediction for Space Shuttle reentry trajectory up to altitudes ~ 120 km (at $R \approx 1\ m$).

5. ACKNOWLEDGEMENTS

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