PARALLEL COMPUTATION OF RADIATION TRANSPORT AROUND REENTRY VEHICLE

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Abstract. Numerical investigations of radiation transport problem around a reentry vehicle are carried out using the model of diffusion approximation. The model was coupled with the gas dynamics part including the radiation flux in the energy equation. The quasi gas dynamics equations (QGD) were used to compute gas dynamic flows. A radiation database has been developed in order to use the values of the absorption coefficients for the problem. The domain decomposition technique was applied for parallelization of the problem.

The goal of this work is to present a parallel finite-difference method for the numerical analysis of radiation influence on gas dynamic flows. The problem has been investigated for the temperature processes 10000K-50000K in axis symmetric *(r-z)* geometry. The radiation transport problem is considered for air[1,2,3]. The numerical schemes have been realized as program complex. The program complex allows to set an object of geometry and environment properties, to generate meshes and to make calculations. Computing meshes are locally condensing, unstructured, are based on triangular and rectangular cells.

1. NUMERICAL METHODS

1.1. Initial conditions

The diffusion approximation method has been applied in this work to obtain numerical solutions for radiation transport around a reentry vehicle. To compute gas dynamic parameters the QGD system of equations was implemented[4,5,6]. Initial conditions of the free stream are shown in Table 1.

The described technique is used to modeling of two dimensional flows around of the body with 0.5035 m radius. It is the maximal radius of the body. For calculations the body of two shapes was used: bullet and egg. The computational domain is shown in Figure 1.

Background gas temperature $T_0(K)$	266
Temperature of head shield $T_1(K)$	2000÷3300
Pressure $p_0(Pa)$	4.3*10
Density $\rho_0(kg/m^3)$	5.63*10^{-4}
Characteristic size of problem $L(m)$	5.035*10^{-1}
Mach number M_∞	12; 24

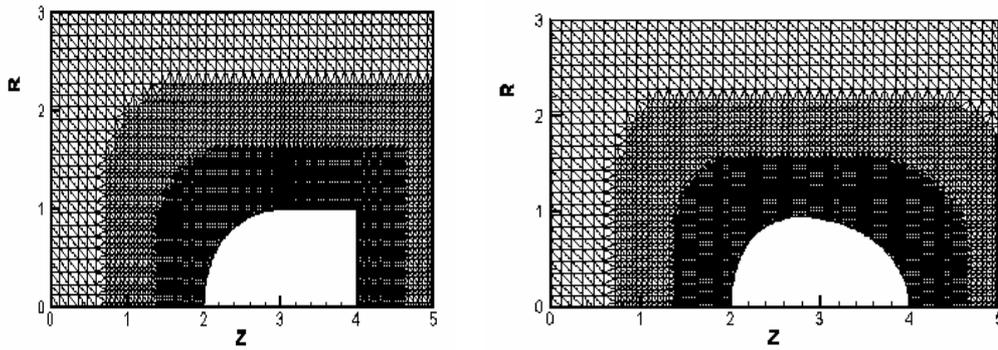Table 1. The initial conditions of the free stream.

Figure 1. The schemes of the computational domains

1.2. The quasigasdynamics equations

Most of existing programs for calculation of viscous flows are based on Navier-Stocks equations system. The original mathematical model proposed in this article utilizes quasi gas dynamics equations system. QGD system is noted for extra dissipative components with small parameter as coefficient. The use of extra dissipation allows us to improve calculation algorithms essentially. The QGD system of equations differs from famous Navier-Stocks system by extra dissipative additives that improve stability of numerical algorithm. This feature makes radiation gas dynamics numerical algorithms based on QGD equations essentially advantageous[5,6]. The fact is that radiative additions to the energy equation cause extra instability factor. In this case there is no abilities to effectively normalize numerical solution in Navier-Stocks equations system. In contrast, the QGD system of equations includes a natural normalizer.

The QGD system used for flow modeling consists of mass, impulse and energy balance equations respectively. With the usual notations, the general form of QGD system is written as (1)-(3):

$$\frac{\partial}{\partial t}\rho + \nabla_i \rho J^i = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\rho u^k + \nabla_i J^i u^k + \nabla^k p = \nabla_i \Pi^{ik} \tag{2}$$

$$\frac{\partial}{\partial t}E + \nabla_i \frac{J^i}{\rho}(E+p) + \nabla_i q^i = \nabla_i(\Pi^{ik} u_k) \tag{3}$$

with $E = 0.5\rho u^i u_j + \varepsilon$. Here ρ is the density, $\vec{u}$ the velocity vector, $p = \rho RT$ the pressure, R is the perfect gas constant, T is the temperature, $\varepsilon = p/(\rho(\gamma - 1))$ the internal energy, γ and is the specific heat ratio. The system (1) – (3) can be closed by the following relation for the mass flux vector J^i, the shear-stress tensor Π^{ik}, and the heat flux vector q^i (4)-(**7**) next example is a multi-line equation:

$$J^i = J^i_{NS} - \tau(\nabla_j(\rho u^i u^j) + \nabla^i p) \,,\, J^i_{NS} = \rho u^i \tag{4}$$

$$\Pi^{ik} = \Pi^{ik}_{NS} + \tau(u^i(\rho u^j \nabla_j u^k + \nabla^k p) + \mathsf{g}^{ik}(u_j \nabla^j p + \gamma p \nabla_j u^j)) \tag{5}$$

$$\Pi^{ik}_{NS} = \mu(\nabla^k u^k + \nabla^i u^k - (2/3)\mathsf{g}^{ik}\nabla_j u^j) + \eta \mathsf{g}^{ik}\nabla_j u^j, \tag{6}$$

$$q^i = q^i_{NS} - \tau\rho u^i(u^j \nabla_j \varepsilon + p u_j \nabla^j(1/\rho)) \,,\, q^i_{NS} = -k\nabla^i T \tag{7}$$

here the coefficients μ and κ are the viscosity and heat conductivity coefficients respectively, g^{ik} is the metric tensor, τ is the Maxwell's relaxation time, equal to $\tau = \mu/p$.

QGD equations are obtained by a kinetic approach. The Boltzman equation is integrated with collision invariants, using a special variant of serial expansion for the distribution function[5]. For stationary flow, the dissipative terms in the QGD equations have the asymptotic order of $O(Kn^2)$ for $Kn \to 0$.

1.3. Diffusion approximation

Modeling of the radiation transfer is realized by the diffusion approximation model[4,7]. The radiation transfer equation is replaced by two equations: the exact equation of continuity for the radiation and the approximate equation that connects the flux and the density of the radiation. The second equation is obtained with assumption of the angular isotropy of the radiations field. The system of diffusion equations must be evaluated many times for different wavelengths. The diffusion approximation model was coupled with the gas dynamic model by the including the radiation flux in the energy equation.

The system of diffusion equations is written as (8)-(9):

$$div\vec{W}_\nu + \chi_\nu c U_\nu = \chi_\nu c U_{\nu p}, \tag{8}$$

$$\frac{c}{3}grad U_\nu + \chi_\nu \vec{W}_\nu = 0 \tag{9}$$

Here $\vec{W}_\nu$ is the radiation flux for the frequency ν or for the wavelength $\lambda = c/\nu$, c is the light velocity, χ_ν is the absorption coefficient for the frequency ν, U_ν is the

spectral density of radiation and U_{vp} is the spectral density of the equilibrium emission.

We have considered the hypotheses of the local thermo dynamical equilibrium. Under this assumption the absorption coefficient at any given frequency is related to the emission coefficient via the black-body function evaluated at the local temperature. The diffusion equations are evaluated for some number of the frequency groups. The total radiation flux is obtained as the sum of the radiation fluxes for each frequency in the form: $\sum_v \vec{W}_v$. At the last step the energy equation is recalculated in the form (10):

$$\partial E / \partial t = div\vec{W}$$ (10)

Thus we obtain parameters of gas dynamic flow fields with radiation.

2.   DATABASE

The data from book by L.M Biberman et al. "The optical characteristics of hot air" have been used for design database. A radiation database has been developed in order to use the values of the absorption coefficients in parallel code.

The diffusion equations were evaluated for 600 frequency groups. Spectral interval is $250 cm^{-1}$ - $150000 cm^{-1}$. In the database the spectral step is $250 cm^{-1}$. Temperature step is 500K. The database supports approximately 10^5 meanings of the absorption coefficients. We can call data from database and use them in our parallel code. Well known limitations of the absorption coefficients values for the diffusion approach are checked (11):

$$\frac{l_v}{L} \langle\langle 1 \quad , l_v = \frac{1}{\chi_v}$$ (11)

here L is characteristic size of the problem, χ_v - absorption coefficient for the frequency v.

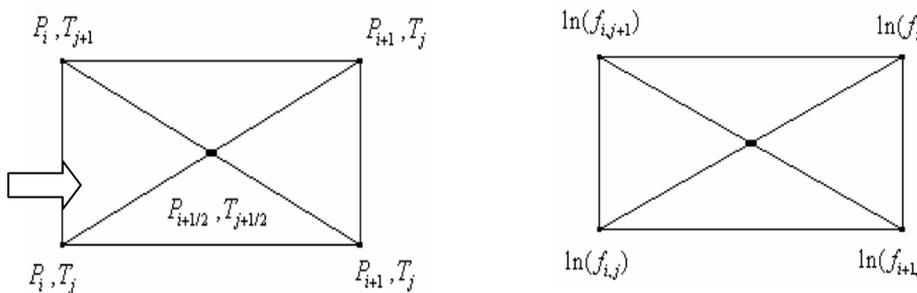We should consider these restrictions for uniformly loading of the computing system.

Figure 2 The scheme of the approximation

To obtain intermediate values of the absorption coefficients the two dimensional logarithmic approximation has been used. The scheme of the approximation is shown in Figure 2. The database has 18 meanings for temperature and 10 meanings for pressure. At certain temperatures some groups do not take part in calculations.

We consider the approximation on *(P,T)* cell. If we have the value of the absorption coefficients inside of a cell (P_i, T_j), (P_i, T_{j+1}), (P_{i+1}, T_{j+1}), (P_{i+1}, T_j), the following algorithm is applied. For approximation we shall take logarithms from the absorption coefficients in nodes of the cell. The absorption coefficient is designated as $f_{i,j}$. Value

for $(P_{i+1/2}, T_{j+1/2})$ is equals $\frac{1}{4}\left(\ln(f_{i+1,j}) + \ln(f_{i,j}) + \ln(f_{i+1,j+1}) + \ln(f_{i,j+1})\right)$.

Here we use linear interpolation on the triangle.

For air radiation O_2, N_2, NO, N_2^+ are considered.

3. PARALLELIZATION AND LOAD BALANCING

Calculations were carried out using high performance multiprocessing systems with the distributed memory. For this purpose parallel algorithm has been developed. The parallel algorithm uses special splitting of computational domain and spectrum. The equations of gas dynamics are solved on a "pipe-line" or "grid" of processors. We use explicit schemes on time and have high efficiency for the gas dynamics block, especially in the case of a grid of processors. It is fair both for a rectangular computational grid, and for an irregular triangular computational grid.

The equations of radiation transfer are solved using linear splitting of spectrum. The diffusion equations consume more than 95 % of total calculation time. Therefore their computing should be the most effective.

Each group contributes to radiation field only if the temperature and pressure of environment fall within some range of values. In particular, it has been discovered, that at temperatures less then 600-700K any of groups does not work. i.e. influence of radiation is not essential. Therefore, we determine whether or not to include in calculation particular group. As a result the volume of calculations of the radiation block substantially diminishes. In particular, with small steps the number of groups can decrease from 600 to 100 or less. To take this fact into account, dynamic distribution of groups on processors and application of dynamic loading algorithm of processors is necessary. In addition the iterative procedure of the solution of the group equations does not guarantee their solution by an identical number of iterations. As a result, some equations converge for 5-10 iterations and others for 50-100. It depends on a spectrum of resulting algebraic problems. Distribution of groups on processors is carried out being based on their computing difficulty. Efficiency of load balancing of processors is approximately 10 % at the initial stage of calculations (at low temperatures), and approximately 60-80 % during next periods.


4. SIMULATION RESULTS AND CONCLUSIONS

The examples of calculations on a hybrid grid (containing rectangular and triangular cells[9]) for Mach number 12 are presented. In the figures 3, 4 stationary distributions of density, Mach number, temperatures, pressure and density of radiation are shown. From the figures clearly, that the zone of increased radiation, in this case almost entirely, is located behind the front of a shock wave, and only a small part of radiation energy emitted in front of a body. This is caused by extreme rarefaction of gas in the top layers of atmosphere.

Some general conclusions of the presented problem entail massive calculations. The high performance parallel computing system has been used here to reproduce radiation heat transfer within reasonable computation time.
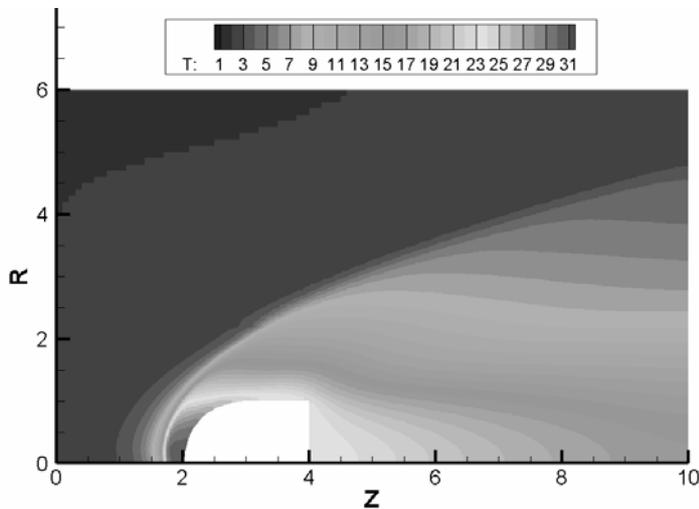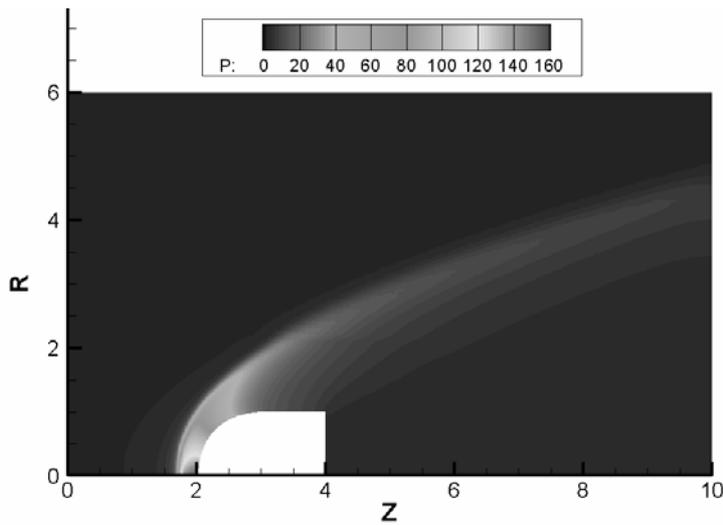
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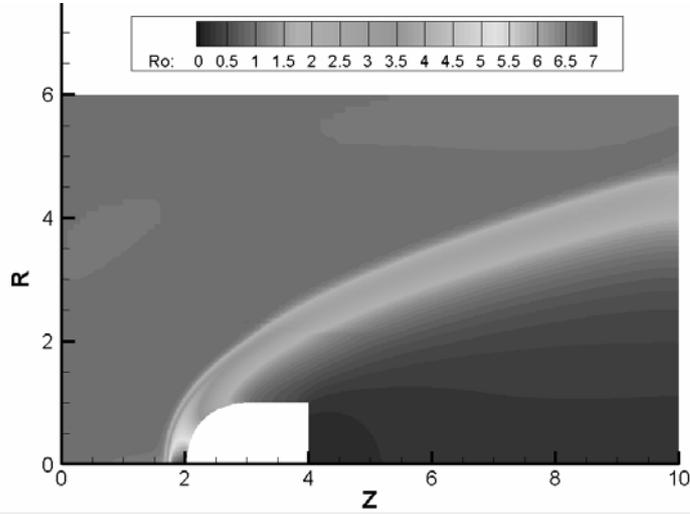
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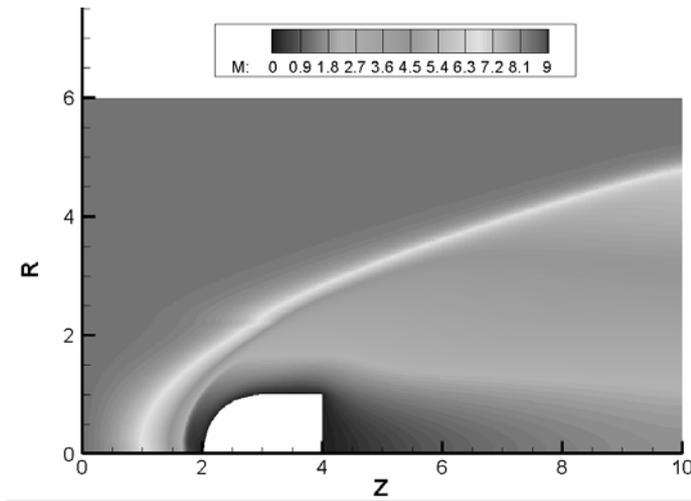
c)

d)

Figure 3. Steady state distribution of gas dynamics parameters: density (a), Mach number (b), temperature (c), pressure (d)

The parallel realization is based on the geometrical parallelism principle. Increased efficiency can be obtained by three dimensional parallelization of the radiation part: two dimensional parallelization on the computational domain and parallelization in accordance with the radiation frequency groups.

The database allows us to compute values of the absorption coefficients and to use them in parallel code. This processing solves the problem of static load balancing and dynamic load balancing.

Numeric results show that the radiation heat transfer processes are of primary importance for accurate prediction of gas dynamics fields around reentry vehicles. Difference between results with and without radiation makes clear the necessity of radiation calculations.

Computational problems for the near future include: including additional gas parameters; describing chemical reactions near the body; describing heat transfer from body surface; describing transformation and evaporation of material on body surface; and the investigation of 3D problem on irregular tetrahedral grids.
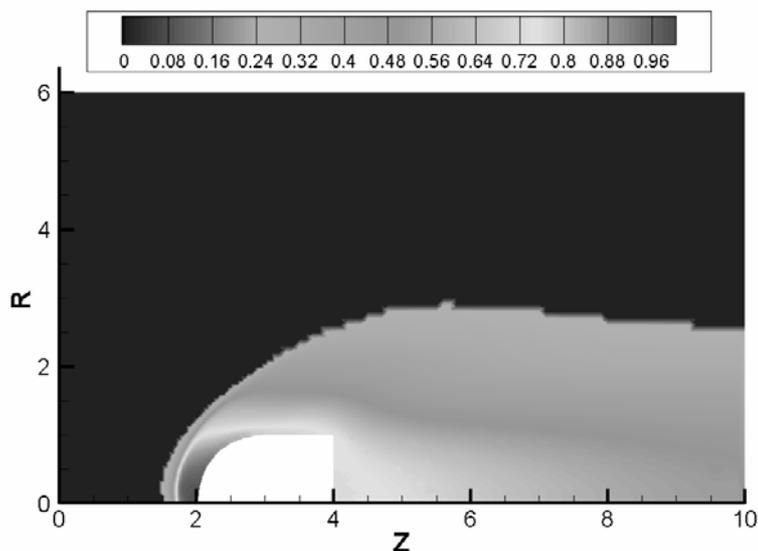

Figure 4. The scheme of the approximation

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